Organizations and Overlapping Generations Games: Memory, Communication, and Altruism

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Abstract

This paper studies the role of memory and communication in overlapping generations (OLG) games between ongoing organizations. In each organization, each individual, upon entry into the game, replaces his predecessor who has the same preferences and faces the same strategic possibilities. An individual has no prior memory — that is, he does not directly witness the events that occur before his tenure. Instead, each individual relies on information about the past from his predecessor via cheap talk. This paper highlights the role of communication as a surrogate for memory.

It has been shown elsewhere that Folk Theorems hold in OLG games with long enough lived individuals who can perfectly observe the past. However, the Folk Theorem fails for many games when individuals have no prior memory. We show that for OLG games without prior memory but with costly communication, a Folk Theorem holds only when there is some altruistic link between cohorts in an organization. Our main result asserts that if communication costs are sufficiently small, or if altruistic weights on successors are sufficiently large, then a *strongly stationary* Folk Theorem (i.e., equilibrium payoffs are time invariant) obtains if a manager's message is public information. The equilibria in this Folk Theorem require a special form of intergenerational sanctions. In these sanctions, punishment is sometimes carried out long after both victim and perpetrator have left the game. Without this special structure, altruism may in fact destroy cooperation when it would otherwise be possible.

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1 Introduction

Societies and organizations are ongoing concerns. They may continue on as long as they are politically or financially viable. Hence, relationships *between* societies or *between* organizations are also ongoing. In stationary environments, these relationships may resemble infinitely repeated games. In such games, "cooperative" outcomes that are mutually superior to one shot play are attainable if the players are patient enough.

However, games between societies or between organizations are not games between individuals. Though they may "live on" indefinitely, societies and organizations are managed by individuals with shorter and finite tenures. These games therefore have a dynastic bent. Individuals within any one "dynastic organization" enter, play the game, and are eventually replaced. If the entry times of individuals happen to be sequenced across organizations, then the game is played between overlapping generations of individuals.

Such situations are distinct from standard, infinitely repeated games. For one thing, an individual in one organization may face different generations of another organization. For another, no person can directly witness events which occurred before he arrives. Consequently, each person's direct memory is limited to his own experiences. These two attributes together have important strategic consequences since they imply that an individual's play against an "old" member in another organization will not be "remembered" by the incoming "young" member in the same organization.[1]

Consider, for example, how a firm's manager might react to a rival's price fixing proposal, knowing that his reaction will not be observed by the incoming manager of the rival firm. Alternatively, consider how the leader of one political party would react to a proposed power-sharing arrangement if the proposal was made by the outgoing leader of the rival party. In reality, past actions are cataloged and conveyed to new generations of managers or leaders. Because past events are not witnessed directly, the past is communicated to new leadership through "cheap talk" communication by the present leadership. These intergenerational messages often serve as a surrogate for memory.

While the presumption that individuals have no "prior memory" may seem quite reasonable, existing models of overlapping generations environments typically assume the contrary. That is, it is commonly assumed that the history of play is either perfectly observed or is costlessly verified by all the players. Also, it is commonly assumed that players care only about their own payoffs in their lifetime. When this is the case, Salant (1991), Kandori (1992), and Smith (1992) have proven various versions of a Folk Theorem for overlapping

[1] Both Hammond (1975) and Cremer (1986) examine models of organizations. Their models have in common with the present analysis the first attribute (overlap between managerial tenures across organizations), but not the second (limited observability).

generations games. The most common statement of this result is that any individually rational payoff of the game is approximated to an arbitary degree by a subgame perfect equilibrium when (1) individuals are patient and live long enough, and (2) the length of overlap between individual's lives is long enough.

This Folk Theorem serves as a useful benchmark for the present paper which studies the role of communication as a surrogate for memory in repeated games between organizations. In particular, we examine an overlapping generations environment with two "organizations". Each organization is characterized by a succession of managers, each of whom remains in the organization for some fixed length, T, then exits, only to be replaced by a successor who has the same preferences and faces the same strategic possibilities. We assume that each manager's memory is limited to events during his lifetime. We examine the role of communication between generations.

While the main analysis will be conducted in the sequel, it is useful to illustrate how communication plays a surrogate role in sustaining cooperation in the absence of prior memory. Consider an infinite repetition of the Prisoners' Dilemma game in Table 1 played between two organizations. Each organization is represented by a sequence of finitely lived managers. Each manager in Organization 1 (resp. 2) begins his term in the beginning of an odd (resp. even) period, plays the stage game twice, and then is replaced by a successor.[2] As in the standard OLG model, each person's utility is limited to events which occur during his lifetime. Assume this utility is the sum of the payoffs in the two periods.

		Player 2	
		C	D
Player 1	C	3, 3	0, 4
	D	4, 0	1, 1

Table 1: Prisoners' Dilemma

Clearly, in this setting, there is no equilibrium in which a manager in his terminal year in the organization will cooperate with the rival organization. The question, therefore, is whether or not there is some equilibrium in which "young" managers choose "C".

If, as in the standard model of overlapping generations repeated games, individuals observe and remember everything that happened in the past (full memory), then there is a subgame perfect equilibrium in which everyone cooperates when young. The outcome path of this equilibrium is illustrated in Figure 1. To support this path, consider the following strategies. If someone in Organization 1 deviates, he is punished by the next person in

[2] The only exception is the first manager in line in Organization 2, who lives for three periods.

Periods $\cdots$	t	$t+1$	$t+2$	$t+3$	$t+4$	$t+5$	$\cdots$	
Organization 1 $\cdots$	C	D	C	D	C	D	$\cdots$	
Organization 2 $\cdots$	D	C	D	C	D	C	D	$\cdots$

Figure 1: OLG play-path

Organization 2, who switches her action from "C" to "D". With full memory, the punisher is identified as a "police person," and so she will not be punished by the next manager in Organization 1; instead, play reverts to the original equilibrium path. The same punishment scheme is invoked if someone in Organization 2 deviates when she is young. Under this strategy profile, no manager has an incentive to deviate to "D" when young since he obtains a one-shot gain of one but loses three in the following period. Also, no manager has an incentive to deviate by refusing to punish the perpetrator since taking "D" is better than taking C when no subsequent punishment is imposed.[3]

Now, relax the full memory requirement and suppose that no one observes events prior to one's entry date. Then even partial cooperation cannot be sustained unless we have another mechanism which replaces memory. The reason is simple. The young manager in one organization could not observe her rival's actions when her rival was young. But since she cannot condition her current action on the older rival's past actions, no punishment is possible. The logic of this 2-period OLG example extends to situations with long but finite period lifetimes. It is shown that the equilibrium payoffs are, in fact, restricted to those of a finitely repeated game whose duration coincides with the length of overlap of players' lifetimes. In Prisoner's Dilemma, for example, this result implies that only the one-shot equilibrium payoff is sustainable.

Next, suppose that one's managerial predecessor can send a costless message, either "good" or "bad", to her successor, which is also observed by the opponent. Then the predecessor can tell her successor whether the opponent followed the "norm" or not. The message is used by the successor to either reward or punish the opponent. Since individuals

[3] To completely specify the strategy profile, we may assume that any other unilateral deviation is simply ignored. It is easy to see no agent has an incentive to deviate in that case, either.

in their terminal year have no incentive to lie to their successors, the (partially) cooperative equilibrium can again be sustained.

Unfortunately, the incentive to lie is as strong as the incentive to be truthful. Thus, if there is a slight difference in the cost of sending messages, then the predecessor always sends the cheapest message (normally "no message" is the cheapest).[4] Even with an arbitrarily small (or lexicographic) cost of communication, the "truthful" equilibrium vanishes, and only babbling equilibria generally exist. Hence, the world with costly messages looks a lot like the "memoryless" world with no communication.

One way that organizations might escape this stark result is to introduce some amount of intergenerational altruism. While outgoing managers of firms are not obviously altruistic, incentive contracts often include severance packages containing company stock and various contingency clauses. Such contracts are structured to induce longer run preferences for managers. Whether or not intertemporal altruism, induced or otherwise, can help depends largely on how coordination across organizations utilizes the altruistic links.

On the one hand, altruism in even the smallest amounts can destroy cooperation when it is otherwise possible. We provide some natural examples in Prisoner's Dilemma games which show that the smallest amount of altruism eliminates equilibria that sustain cooperation even when communication is costless. To illustrate the point, consider the "grim-trigger"-type strategies in the PD game. According to this strategy profile, once a "D" is detected by a young manager, this manager sends the "bad" message, which induces all others to choose "D" thereafter. This strategy constitutes an equilibrium if there is costless communication and no altruism. If, however, some "old" manager has even the smallest amount of altruism, then even if she is victimized by the deviation, she is nevertheless better off by lying and sending "good" rather than "bad". For if she signals "bad", then all successors (whom she cares about) in her organization will get only 1 on average, while if she sends "good", then all successors receive equilibrium payoff of 2 on average. She will therefore "whitewash" the deviation, thus providing incentives for bad behavior in the rival organization.[5] Consequently, the grim-trigger strategy profile is not an equilibrium when altruism is present.

On the other hand, a little bit of altruism may help if the right kinds of sanctions are used. Our main result asserts a *strongly stationary* Folk Theorem for the environment with two organizations, no prior memory, costly communication, and intergenerational altruism. Specifically, we show that for each environment, any feasible payoff can be approximated to arbitrary degree by a sequential equilibrium equilibrium profile whose path behavior is

[4]In organizations, sending messages often corresponds to filing reports, which requires effort of those who make them. In this case, providing the default memo may be considered to be the cheapest way of doing the job.

[5]The potential for this kind of "whitewashing" has an effect similar to that of renegotiation in repeated games (see, for example, van Damme (1989)) and is the central theme in Anderlini and Lagunoff (2001).

time-stationary, provided that communication costs are small enough or individuals are sufficiently altruistic (i.e., they place enough weight on future generations within their respective organizations).

We examine the environment where communication is public. We examine the Perfect Public equilibrium of the OLG game when each manager's report to his successor is public information. This setting may be important when relevant information is contained in, say, the firm's prospectus. If, however, reporting or communication is internal to the organization, then it is shown that the same mechanism does not work to sustain cooperation between organizations.

These results are of interest because they suggest that even cheap talk communication is an effective surrogate for memory. In light of our previous remarks about the potential harm of altruism, it is just as important that the equilibria must assume a special structure to sustain cooperative outcomes. For example, the messenger must be indifferent over all the messages that she might send in any equilibrium continuation. Anyone called upon to truthfully report a deviation can neither benefit by nor be harmed by her report. As it turns out, in order to sustain this "neutrality" of the sender, sactions must be "generation skipping": punishments must sometimes be carried out long after both victim (sender) *and* perpetrator have left the game.

The paper is ogranized as follows. Section 2 introduces the basic model. We compare outcomes under full memory to those without prior memory. Section 3 evaluates the role of communication. First, the case of costless communication is considered. This contrasts starkly with the case of costly communication. Section 4 examines the role of altruism across generations, and proves the main result. We spend considerable time motivating the special structure of the equilibrium sanctions. Section 5 concludes by examining some natural extensions and by relating the present work to existing literature. We distinguish the present paper from recent work by Bhaskar (1998), Kobayashi (2000), and Anderlini and Lagunoff (2000) which also examine limited observation and, in the last two, communication in "dynastic" frameworks. Section 6 is an Appendix containing the proof of the Folk Theorem.

2 The Benchmark Model

2.1 Stage Games

In each period of the dynamic game which is introduced later, the identical stage game is played by two agents. It is given by

$$G = \langle (S_1, S_2), (u_1, u_2) \rangle,$$

where for each $i = 1, 2$, S_i is the finite set of agent i's pure strategies, and $u_i : S \equiv S_1 \times S_2 \to \mathbb{R}$ is his payoff function. A mixed strategy by Player i is denoted by σ_i where $\sigma_i \in \Delta(S_i)$. The payoff functions u_i's are extended to the mixed strategy space in a standard manner, i.e., for $\sigma = (\sigma_1, \sigma_2)$, $u_i(\sigma) = \sum_{s_1 \in S_1} \sum_{s_2 \in S_2} \sigma_1(s_1)\sigma_2(s_2)u_i(s_1, s_2)$. For $j \neq i$, we write (σ_i, σ_j) to mean (σ_1, σ_2) and so on whenever there is no confusion. A payoff pair $v = (v_1, v_2) \in \mathbb{R}^2$ is said to be *feasible* if there exists a vector $\lambda \in \Delta(S)$ such that $(v_1, v_2) = \sum_{s \in S} \lambda(s)(u_1(s), u_2(s))$ holds.

Given G, the minimax or *individually rational* payoff of agent i is defined. First, let

$$\underline{\sigma}_j^i = \arg\min_{\sigma_j} \max_{\sigma_i} u_i(\sigma_i, \sigma_j).$$

Then let

$$\underline{\sigma}_i^i = \arg\max_{\sigma_i} u_i(\sigma_i, \underline{\sigma}_j^i).$$

Finally, let $\underline{u}_i = u_i(\underline{\sigma}^i) = u_i(\underline{\sigma}_i^i, \underline{\sigma}_j^i)$. Let $V \subset \mathbb{R}^2$ be the set of pairs of feasible payoffs which are strictly greater than respective individually rational payoffs.

2.2 The Overlapping Generations Game

Here we define the benchmark model. Time is discrete, and the horizon is infinite. Assume that each i $(i = 1, 2)$ indexes an entire lineage of individuals. We refer to these lineages as *organizations* and let i denote the index for organization i. The two organizations face off to play a repeated game. Individuals in each organization are assumed to live for T periods where T is assumed to be an even integer for convenience. In a given organization, an individual is "born" at some date t and T periods later, this individual exits or "dies" and is subsequently replaced by another individual with the same preferences and strategic possibilities.

Significantly, we assume that the replacement process between the two organizations is not synchronized. Specifically, we assume that half-way though an individual's lifetime,

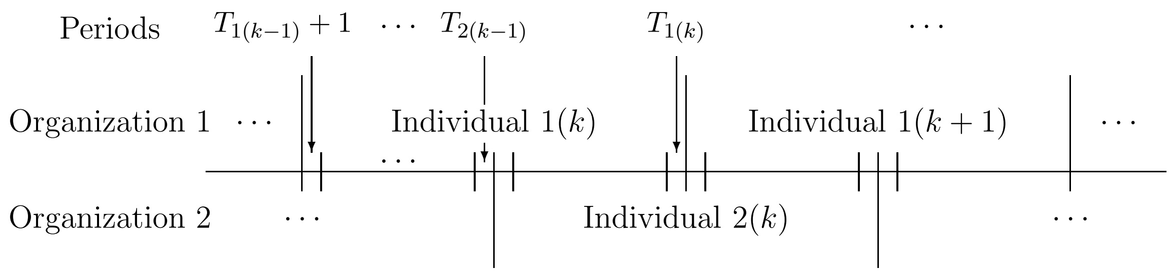

Figure 2: OLG game

a replacement occurs in the other organization. This lack of synchronicity captures the overlapping nature of the game. The fact that an individual plays against both "father" and "son" in the rival organization will prove consequential.

We let $i(k)$ denote the kth individual from the ith organization. The entire set of players is therefore denoted by $\{1(k), 2(k)\}_{k=1}^{\infty}$. For $k = 1, 2, \ldots$, Player $1(k)$ is born in the $((k-1)T+1)$th period, and retires in the $T_{1(k)} \equiv (kT)$th period. Player $2(1)$ is born in the first period, and retires in the $T_{2(1)} \equiv (T+T/2)$th period, while Player $2(k)$ ($k = 2, 3, \ldots$) is born in the $((k-1)T + T/2 + 1)$th period, and retires in the $T_{2(k)} \equiv (kT + T/2)$th period. As defined above, $T_{i(k)}$ is the date of Player $i(k)$'s retirement. Let $T_{1(0)} = T_{2(0)} = 0$. Then Player $i(k)$'s birth date is given by $T_{i(k-1)} + 1$. Figure 2 illustrates this environment. In the following, when we refer to the life-span of the players, we ignore the special status of Player 2(1). This should cause no confusion.

The obvious application of this setup is to firms or other organizations controlled by managers. Each manager's tenure is short, while the organization lives on in the dynastic setting. For now, we assume that preferences of currently lived individuals do not extend beyond their lifetimes (e.g., a manager does not care what happens to the firm after he leaves it). This assumption will be modified later on. Hence, each player tries to maximize his own average payoff across his life time. It is assumed that there is no discounting. A player who receives a sequence of stage payoffs $(u^1, u^2, \ldots, u^T)$, has a payoff given by

$$\frac{1}{T}\sum_{t=1}^{T} u^t.$$

In what follows, we characterize payoffs sustainable by equilibria of the game. In the case where players have full memory, these equilibria are subgame perfect equilibria. However, when memory is limited, the natural extension of subgame perfection is captured by the sequential equilibrium concept.

2.3 The Perfectly Observable Past

This subsection presents a benchmark situation which has been studied elsewhere. We assume that the full history of play is available to each individual upon entry into the game. Formally, for $t = 2, 3, \ldots$, a period t *behavior history* is a tuple $h^t \equiv (\sigma^1, \sigma^2, \ldots, \sigma^{t-1})$ of action profiles observed by time t.[6] The null history is denoted by h^1. The set of period t behavior histories is given by H^t, and let $H = \cup_{t=1}^{\infty} H^t$ denoting the collection of all (finite) behavior histories.

For simplicity, we will assume that individuals have access to a public randomizing device so that they may use correlated strategies in each period. However, we omit the notation which carries along the realization of this device in each person's behavior strategy. Since each individual in a organization conditions his behavior on a history whose path length is distinct from that of any other individual in the organization, there is no loss of generality in defining a behavior strategy of player $i(k)$ to be a function $f_i : H \to \Delta(S_i)$, which is independent of index k. Let $f = (f_1, f_2)$. Let $f(h) = (f_1(h), f_2(h))$. A profile, f, of strategies is a *Subgame Perfect equilibrium (SPE)* of the OLG game if, for each $i(k)$, $(i = 1, 2,\ k = 1, 2, \ldots)$, and after every history h^t with $t = T_{i(k-1)} + 1, \ldots, T_{i(k)}$, strategy f_i maximizes

$$\frac{1}{T} \sum_{\tau=t}^{T_{i(k)}} u_i^{\tau}(\ f(h^{\tau})\ )$$

given $f_j,\ j \neq i$. Finally, we will say that a payoff v is *sustained* by a stationary SPE profile f if

$$v_i \ = \ \frac{1}{T} \sum_{\tau=T_{i(k-1)}+1}^{T_{i(k)}} u_i^{\tau}(\ f(h^{\tau})\ )$$

for each individual $i(k)$.[7]

The following Folk Theorem applied to OLG games has been proved by Kandori (1992) and has been extended to more general settings by Smith (1992).

Theorem 1 *(Kandori (1992)) Given a stage game G, consider an OLG game with perfect*

[6] We assume here that players' mixed strategies are observed. However, we could just as easily relax this requirement and assume observability of only pure action profiles.

[7] Stationary SPE have been defined as SPE which every individual in the same organization has the same outcome path. A weaker requirement has been proposed by Prescott and Rios-Rull (2000) for stationary (but not necessarily repeated) OLG games.

observation. For all $\epsilon > 0$ and all $v \in V$, there exists $\underline{T}$ such that for all $T \geq \underline{T}$, there exists $v' \in V$ such that $|v - v'| < \epsilon$ and v' is sustained by a stationary SPE of this OLG game.

Kandori (1992) provides the essential intuition: equilibria in OLG games may be understood as a sequence of finitely games repeated games with bonding. Each individual receives a contingent reward at the end of his life in exchange for his cooperation earlier on. A complete version of the proof may also be found in that paper.

2.4 The Unobservable Past

Suppose now that each player cannot observe the history of play before he joins the game. This is a natural assumption since, by definition, this player was not around to directly witness of events that occurred before his entry. In such a game, Player $i(k)$'s strategy is conditioned only on his *personal history.* The personal history of individual $i(k)$ consists of the path of events that occurred after $i(k)$'s entry. Formally, for any integer $t > T_{i(k-1)} + 1$, the personal history of $i(k)$ at date t is denoted by

$$h^t_{i(k)} \equiv (\ \sigma^{T_{i(k-1)}+1},\ \sigma^{T_{i(k-1)}+2}, \ldots, \sigma^{t-1}\ ).$$

If t is the entry date of $i(k)$, i.e., if $t = T_{i(k-1)} + 1$, then $i(k)$ conditions on the null history h^1 so that $H^{T_{i(k-1)}+1}_{i(k)} = \{h^1\}$.[8] For $t \geq T_{i(k-1)} + 1$, let $H^t_{i(k)}$ denote the set of all date t personal histories , and let

$$H_{i(k)} \equiv \cup_{t \geq T_{i(k-1)}+1} H^t_{i(k)}$$

denote the set of all personal histories for $i(k)$. A behavior strategy now maps only from the individual's personal histories: let $g_{i(k)} : H_{i(k)} \to \Delta(S_i)$ denote the behavior strategy of individual $i(k)$ when only events during $i(k)$'s lifetime are observable to him. We now define a *sequential equilibrium* profile $g = \{(g_{1(k)}, g_{2(k)})\}$ in the usual way: each individual plays a best response after very personal history, and players' beliefs are consistent.[9]

We observe that, under the stated hypothesis, if players lack memory before their entry, then the Folk Theorem property fails to hold in some games. Consider, for example, the OLG game of Prisoners' Dilemma in Table 1 in the Introduction. In this game, Player $1(k)$ has an incentive to take D in the last period of his life, i.e., $g_{i(k)}(h^{T_{i(k)}}) = 1_D$ where 1_D is the mixed strategy that places full mass on D. Since $T_{i(k)}$ is the last period of his life, he does not have to worry about future punishment. In addition to this behavior, Player $2(k)$ has an incentive to take D in the (kT)th period. This is because this defection is not observed by

[8] Our notation allows for histories that exceed the length of individual $i(k)$'s lifetime. This is merely for notational convenience. It clearly has no consequence for this individual's behavior in the game.

[9] Consistency requires that players' beliefs be limits of sequences of beliefs that (1) place positive probability on all information sets, and (2) follow Bayes Rule.

the next generation of the opponent, and therefore, despite that she will live for another $T/2$ periods, she does not have to worry about future punishment, either. Thus, in the (kT)th period of Player 1's lifetime, neither player will take C.

Once we establish this result, we simply unravel their strategies. In the $(kT-1)$th period, they have no incentive to take C since neither punishment nor reward is expected in the future periods, and so on. Hence, the only equilibrium in this game with no observability is the one in which all the players in both organizations repeat D forever.[10]

Generally, the set of sequential equilibria coincides with that of a finitely repeated game whose duration is the same as the length of "overlap" between players.

Theorem 2 *Given a stage game G, consider a (two-player) OLG game in which no individual observes play before he enters. Then in any sequential equilibrium (stationary or not), the play beginning in periods $T_{1(k)}+1$ (resp. $T_{2(k)}+1$) and ending in periods $T_{2(k)}$ (resp. $T_{1(k+1)}$) , $k=1,2,\ldots$, coincides with that of an SPE of a $T/2$-period finitely repeated game.*

It is left unstated, but clear that for $k=0$, the play for the initial generation coincides with a T-period finitely repeated game.

Proof Consider an individual, say $1(k)$, currently living when a new individual $2(k)$ enters. These two individuals share $T/2$ periods of life together. Since $2(k)$ cannot observe history prior to her entry, her strategy in period $T_{2(k-1)}+1$ is $g_{2(k)}(h^1)$. That is, her strategy is non-contingent. Clearly, individual $1(k)$ cannot gain by tailoring his strategy to actions that occur before $2(k)$'s entry date. At the same time, since $1(k+1)$ cannot observe the history of play prior to his entry, $2(k)$ cannot gain by tailoring her strategy to actions that occur before $1(k+1)$'s entry date. Hence, actions from $T_{2(k-1)}+1$ to $T_{1(k)}$ must constitute an SPE of a game whose length is $T_{1(k)}-T_{2(k-1)}=T/2$. $\diamondsuit$

3 Can Communication Help?

This section considers OLG games in which information transmission is available from one generation to the next. At the end of one's life, one disseminates a piece of information, which is received by his successor as well as his opponent. Let M be a message space. We assume that $H\subset M$, i.e., one can describe any history via his message.

[10]The harsh outcome of this particular counterexample can be mitigated by increasing the number of players, each of which enters asynchronously. In this case, the gain to deviating in the penultimate period of one's life is balanced by the punishment one incurs in the last period inflicted by those who observed the prior defection.

The set of histories as well as the sets of personal histories are modified accordingly. In particular, the set of personal histories of Player $i(k)$ at time $t \geq T_{i(k-1)} + 1$ is now given by

$$\bar{H}^t_{i(k)} = \begin{cases} M \times H^t_{i(k)} & \text{if} \quad t \leq T_{i(k-1)} + T/2, \\ M \times M \times H^t_{i(k)} & \text{if} \quad t > T_{i(k-1)} + T/2, \end{cases}$$

where in $\bar{h}^t_{i(k)} = (m, m'; h^t_{i(k)}) \in \bar{H}^t_{i(k)}$ when $t > i(k-1) + T/2$, when m is the message he obtains from his predecessor, and when m' is the message received from his opponent in the middle of his lifetime. The behavior strategy $g_{i(k)}$ is extended to these histories/messages accordingly.

A reporting strategy by an individual $i(k)$ is a function $\mu_{i(k)}$ that maps from histories in the last period of $i(k)$'s lifetime to messages. We write $\mu_{i(k)}(\bar{h}) = m$ to denote that the individual sends message m given history $\bar{h} \in \bar{H}^{T_{i(k)}}_{i(k)}$ (we drop subscripts $i(k)$ on $\bar{h}$ whenever the meaning is clear). A strategy profile is a sequence of behavior and reporting strategies for each individual in each organization:

$$(g, \mu) \equiv \{(g_{1(k)}, \mu_{1(k)}), (g_{2(k)}, \mu_{2(k)})\}.$$

Since the limited observability environment induces a game of incomplete information, it is appropriate to consider sequential equilibria of the game. The notion of *sequential equilibrium* can now be extended to the pair (g, μ).

In the next subsections, we first consider the case in which transmission is costless, then proceed to the case of costly transmission. We show that the implications are starkly different in each case.

3.1 Costless Communication

In this case, an individual who communicates to his successor is clearly indifferent between any pair of messages; for the communicating individual, there is neither cost of sending a message nor is there an altruistic motive favoring certain continuations over others in the game after he leaves. Therefore, he has no incentive not to honestly report the past history including what he observed in his life time as well as the message he received by his own predecessor (unless he is in the first generation).

To see explicitly how communication works in the limited observability world, consider once again the repeated Prisoners' Dilemma in Table 1. In the game with costless communication with $\{C, D\} \subset M$ and a sufficiently large discount factor, there exists a sequential equilibrium in which every player takes a grim trigger-like strategy (g, μ), where, for each individual $i(k)$,

$$
g_{i(k)}(\bar{h}^t) = \begin{cases} C & \text{if } [t \leq T_{i(k-1)} + T/2, \text{ and } \bar{h}^t = (m; h^t_{i(k)}) \text{ contains no } D \\ & \text{except possibly a } D \text{ by the opponent at } T_{i(k-1)} + T/2] \text{ or} \\ & [T_{i(k-1)} + T/2 < t < T_{i(k)}, \text{ and in } \bar{h}^t = (m, m'; h^t_{i(k)}), m' = C, \\ & \text{and } h^t_{i(k)} \text{ contains no } D \text{ after } T_{i(k-1)} + T/2], \\ D & \text{otherwise.} \end{cases} \tag{1}
$$

and

$$
\mu_{i(k)}(\bar{h}) = \begin{cases} C & \text{if } \bar{h} \text{ contains no } D, \\ D & \text{otherwise.} \end{cases} \tag{2}
$$

Roughly, on the equilibrium path, the players play C except in the last period of their tenures and report C. If there is a deviation from this path, they take D and report D. The only exception is when after some deviation, the previous opponent reports C instead of D, in which case they revert back to the initial equilibrium play.[11]

With costless messages, a player can be assured that his rival will be punished in the periods past his lifetime, should his rival defect. More generally, it is clear that the natural extension of Theorem 1 holds for this case:

Theorem 3 *Given a stage game G, consider an OLG game in which (1) each individual cannot observe past play before his entry into the game, and (2) upon his exit, each individual in a organization communicates publicly and costlessly to his successor. Then, for all $\epsilon > 0$ and all $v \in V$, there exists $\underline{T}$ such that for all $T \geq \underline{T}$, there exists $v' \in V$ such that $|v - v'| < \epsilon$ is sustained by a stationary sequential equilibrium with truthful communication.*

Since communication is costless, the result can easily be extended to the case where messages are observable only to the successor within the same organization. While there is no incentive to misreport, it is clear that there is no strict incentive to honestly report the past history. Therefore, with costly communication, no matter how small the cost, the equivalence between a perfectly observable past, and an unobservable one with communication breaks down. This leads us to examine the next case.

[11] This reversion is necessary since after the report C by, say, Player $1(k)$, the only player who knows the past deviation is $2(k)$ who has an incentive to whitewash the past by playing C.

3.2 Costly Communication

If communication is costly, then the Folk Theorem need not hold. This is true even if the cost is arbitrarily small. Suppose that the cost $c_{i(k)}(m)$ for individual $i(k)$ who sends message m enters additively into his payoff. In particular, suppose that there is a unique message m^* for which $c_{i(k)}(m^*) < c_{i(k)}(m)$ for all other m. Message m^* may correspond to the choice of sending no message. Then a predecessor has a strict incentive to send this least costly message. But if this message is always sent, then we are back to the case when the past is unobserved.

4 Can Altruism Help?

If communication is costly, then the only reason an individual might be willing to communicate with his successor is if he has some personal stake in doing so. It is natural to consider the possibility that individuals have a stake in the organization which extends beyond their own tenure. We examine how things change when each individual in the organization cares about his successors. Can a little intergenerational altruism help in facilitating social cooperation? The answer depends on whether the equilibrium can be tailored to the degree of altruism that exists.

4.1 A Theorem and an Example

Let $\delta \in (0,1)$ be a weight that an individual player places on the payoffs of future generations in the organization. Therefore, if the ith organization attains the payoff stream $(u_i^1, u_i^2, \ldots)$, then the weighted average payoff of Player $i(k)$ is now defined to be

$$(1-\delta)\sum_{\kappa=k}^{\infty}\delta^{\kappa-k}\frac{1}{T}\left[\sum_{t=T_{i(\kappa-1)}+1}^{T_{i(\kappa)}} u_i^t - c_{i(\kappa)}(m)\right]. \tag{3}$$

We will say that a payoff v^* is sustained by a *strongly stationary* sequential equilibrium (g^*, μ^*) if for each individual $i(k)$, and all equilibrium path histories, h^*,

$$v_{i(k)} \;=\; u_{i(k)}(\; g^*(h^*)\;)$$

Normally, stationarity for OLG games requires only that each generation's sequence of actions, hence its average payoff, is the same. Strong stationarity must satisfy the additional requirement that each individual's payoff be the same each period. Below, we state and prove a "strongly stationary" Folk Theorem for the game with altruistic preferences given by (3), provided that communication costs are not too large. However, the argument is not so simple as to be able to simply extend previous results. In the Theorem, let c denote the difference between the least costly and second least costly messages.

Theorem 4 *Suppose that the stage game G is of full dimension, i.e., $int(V) \neq \emptyset$ where $int(V)$ is the interior of V. For all $v^* \in int(V)$ and all $\delta \in (0,1)$, and for all $\varepsilon > 0$ such that $c = \delta\varepsilon$, there exists $\underline{T}$ such that for all $T \geq \underline{T}$, v^* is attained by a strongly stationary, (Perfect Public) sequential equilibrium of this OLG game.*

Hence, with the introduction of altruism, the Folk Theorem can once again be salvaged. The following remarks may further clarify the result.

First, notice that, as before, this Folk Theorem requires that individuals' lifetimes be long enough. However, unlike other OLG Folk Theorems, each feasible interior payoff may be attained exactly rather than only approximated. Of course, payoffs on the boundary are only attainable to an arbitrarily close approximation.

Second, the requirement of strong stationarity means that "bad behavior" or terminal period rewards need not be built in to the equilibrium path. Individuals can be required to cooperate throughout their tenure. Of course, the trade-off is that a positive, though possibly small, amount of altruism is required, or, for a given degree of altruism (determined by δ), communication costs must be sufficiently small.

Third, it is commonly known that in any game with private information, equilibria come in two varieties. There are the so-called *Perfect Public equilibria (PPE)*, and then there is everything else. Perfect Public equilibria are sequential equilibria which rely exclusively on public signals. Though the reliance on only publicly available information restricts some potential options for the players, the virtue of PPE is that it is a more attractive social norm for society since all players' inferences about the future play path are always the same. Indeed, when communication is public, the PPE restriction is not so restrictive. The algorithm used to prove our Folk Theorem requires only PPE. We think the construction is "canonical" in the sense that it contains certain features essential to any equilibrium algorithm in a Perfect Public Folk Theorem.

On one level, Theorem 4 may not appear surprising. Since altruism extends the time horizon of individuals in the game, one might think that the addition of altruism can do no damage to an *existing* equilibrium of the game. For example, in a standard, infinitely repeated Prisoner's Dilemma game, the grim-trigger which sustains mutual cooperation at some discount factor δ also remains an equilibrium at higher (more patient) discount factors as well. However, in OLG games, extending the time horizon via altruism need not preserve equilibria.

Again consider the Prisoner's Dilemma game and the grim-trigger-like strategy profile defined by equations (1) and (2) in Section 3.1. When communication is costless, this strategy profile sustains mutual cooperation in Prisoner's Dilemma. With altruistic preferences, however, this strategy profile is *not* an equilibrium.

From this strategy profile, an individual, say, 1(k) has an incentive to deviate in the last day of his opponent's, i.e., 2(k-1)'s life. To see there is such an incentive, suppose that 1(k) takes D at time $T_{2(k-1)}$. Then Player $2(k-1)$'s best response is no longer sending a truth-telling message to her successor. The reason is that if she does, then the subsequent outcome will be the repetition of D, while if she does not and reports C instead, then the subsequent outcome will be C forever. Therefore, this strategy profile is no longer a sequential equilibrium.

Thus, in an equilibrium with $g_{i(k)}$'s intact, it has to be the case that $\mu_{i(k)}$'s be modified so that $\mu_{i(k)}(h) = C$, i.e., each player has an incentive to whitewash the past. But if this is the case, then no sanction prevents a player from deviating to D in the last day of his opponent's life. Once we establish this fact, by induction no one has an incentive to take C at any date. Cooperation is therefore destroyed.

The above example suggests that it is not enough for, say, firms, to induce intergenerational altruism by writing incentive contracts that extend the manager's compensation beyond his tenure. If altruism is to be beneficial, equilibria must have a special structure. Indeed, we think the structure of independent interest. For this reason, we devote considerable attention to describing and motivating requisite incentives more carefully.

4.2 The Equilibrium Algorithm: Logic and Structure

The proof is constructive. An algorithm is constructed in which, for any $v \in int(V)$, produces a Sequential equilibrium that sustains v given the appropriate choices of T and c. Before the formalities, however, it is worth looking at the nature of the incentives that must be accounted for in any equilibrium.

Now consider any game satisfying the hypothesis of Theorem 4. Now fix $v^* \in int(V)$, fix some $\delta \in (0,1)$ and fix an $\varepsilon > 0$. To simplify the discussion, we allow for correlated strategies so that V is convex. Consider some putative equilibrium pair (g^*, μ^*) which sustains v^*. We would like to understand the logic of the critical incentive constraints that support (g^*, μ^*). To do this we group them as follows.

Suppose that at some point in the game, an individual deviates. For each i, P denote an integer which will be defined more precisely later on. Call an individual "old" if he/she has fewer than P periods left in his lifetime. Formally, an individual $i(k)$ is old at date t

if $t > T_{i(k)} - P$. Call an individual "young" otherwise. (Note that this convention differs from many OLG models. For if $T_{i(k)} - T/2 < t < T_{i(k)} - P$ then both players are "young" according to our convention despite that Player $i(k)$ is older than his rival by $T/2$ years.) Now in order for (g^*, μ^*) to sustain v^*, the equilibrium must provide incentives to deter three types of deviations:

- Young individual cheats another young individual
- Young cheats old
- Old cheats young

Consider the first type of deviation: young cheats young. It is not hard to see that "typical" sanctions used in OLG Folk Theorems (which we described in the Introduction) are applicable. Since both individuals are "young" there is plenty of time to punish the perpetrator. As with "typical" sanctions, this entails a "minimaxing" (punishment) phase, and a "reward-the sanctioner" phase.[12]

Providing deterrence for the next two types of deviations is where things get interesting. Consider the second type of deviation: young cheats old. Since, by definition, an "old" individual is at or close to the end of his tenure, he cannot punish the young perpetrator directly. Instead he must rely on his successor in the dynastic organization to carry out the punishment. The punishment takes place in the next generation. Hence, the old individual's message must report the deviation. Although communication is costly, he may still benefit from sending the right message since his altruism gives him a stake in the play of the game by future generations (even if he is not there to witness it!).

But there is a problem. Communication is cheap talk. Hence, if the old individual benefits too much from reporting a deviation, then he will always send this message, *regardless of whether or not a deviation actually occurred*. On the other hand, if his future benefit is insufficient to cover his communication cost, then he will never send this message even if a deviation occurs.

To summarize, an old individual must be indifferent between any of the messages he might send in any equilibrium continuation. However, this incentive constraint now requires some care when the sanctions are constructed. After all, in some cases, punishment makes all individuals worse off (see footnote 12), while in other cases "sanctioning is its own reward." As before, punishment has both a minimax phase, and a second, "reward the sanctioner" phase. The difference is, first, that it begins in the next generation; second, the reward to the sanctioner in the second phase may be worse than the first phase punishment itself.

[12] The latter is required if, for example, $\underline{u}_j > u_j(\underline{\sigma}^i)$, i.e., j's punishment of i makes j worse off than in his own punishment.

Finally, there is a third type of deviation to consider: old cheats young. Deterring this type of deviation is even more problematic. As with the previous case, a messenger must be indifferent between all types of messages he might send in equilibrium. Unfortunately, in this case, the messenger (the old individual) is *also* the perpetrator. It is therefore impossible to both provide incentives to deter his deviation while at the same time provide him incentives to report his act honestly. Consequently, the equilibrium must delay the punishment until the young agent is old enough to report the action at the end of the young agent's tenure. To summarize, the equilibrium builds in an initial "lie" by the old perpetrator, and then requires a delayed punishment $T/2$ periods later, when the young individual himself becomes old and can report the deviation. Naturally, this punishment must be longer than the others in order to compensate for the delay.

This means that when old cheats young, punishment of an old perpetrator is ultimately carried out by the victim's successor against the perpetrator's successor! The sanction therefore has the likeness of a dynastic feud.

4.3 Can Altruism Help without Communication?

One may reasonably ask: since the above Folk Theorem allows for some altruism, is communication even required? We suspect that the answer is literally "no" if there is sufficient altruism, though the Theorem must be weaker in some respects. With sufficient altruism, the model more closely resembles repeated games with private monitoring.

Consider, for example, the case where old cheats young. If an old agent $j(k)$ deviates at the end of his tenure, then rival i can initiate a punishment against an unsuspecting $j(k+1)$ provided that i is indifferent between punishing and not punishing $j(k+1)$. Note that $j(k+1)$ will necessarily interpret i's attempt to punish j as a deviation by i. Consequently, individual $j(k+1)$ will, in turn, punish i. Hence, the continuation payoff for i that initiates punishment against $j(k+1)$ for the sins of $j(k)$ must, after the initial period, appear to punish i. If such a continuation can be constructed, then deviations by old against young can be prevented.

On the other hand, without communication, no similar punishment can be constructed when young cheats old. To prevent cheating by young individual $i(k)$, the equilibrium must build in a "deviation" or reward to $i(k)$ at the end of j's tenure. However, this rules out strongly stationary paths.

We conjecture that an approximate, though not strongly stationary Folk Theorem can be proven with sufficient altruism, no communication, and long enough lifetimes. Of course, when there is both altruism and perfect memory, a strongly stationary Folk Theorem holds as well. Therefore, even with some altruism, communication can still help. Communication gives the added advantage of exactly sustaining any feasible interior payoff by a strongly stationary equilibrium. Sanctions do not need to allow for departures from the reference payoff v^* in any period.

5 Related Literature and Extensions

This paper shows how communication and altruism play critical roles in sustaining equilibria in overlapping generations (OLG) games with a dynastic structure. Strongly stationary Folk Theorem holds for a public communication protocol by using a particular type of equilibrium which utilizes intergenerational sanctions. The model has antecedents in the Folk Theorem literature for OLG games (Cremer (1986), Kandori (1992), Smith (1992), Shepsle (1999)) Unlike these papers, an individual's memory in the current model does not extend beyond his own lifetime, though we do extend one's time horizon via intergenerational altruism.

The role of general informational constraints in OLG games was also studied by Bhaskar (1998). For reasons similar to those in our Prisoner's Dilemma example above, he showed that Pareto improving transfers in a 2-period consumption-loan smoothing OLG game are not sustainable in pure strategy equilibria if players have finite memory of past play. However, if players can observe at least one period of past play (before their arrival), then optimal transfers are sustainable in mixed equilibria.

Our informational constraints are more severe than Bhaskar's mixed strategy "Folk Theorem" allows. Hence, instead of some (albeit limited) memory, we utilize cheap talk communication as a vehicle for passing on information. This role of communication has also been examined in the literature on repeated games with private monitoring.[13]

A more closely related model is examined independently by Kobayashi (2000). Like the present work, he also proves a Folk Theorem for OLG games without individual memory and with private communication within organizations. Kobayashi's model differs somewhat from ours in that it induces altruistic links by allowing overlapping structure within as well as across organizational units. Because preferences over the future outcomes have a finite "lifetime" in that model, the equilibria are not strongly stationary and generally look different than ours.

Another related model is found in a model by Anderlini and Lagunoff (AL) (2001) who examine communication in dynastic repeated games. These are games between dynastic entities in which one-period lived individuals can communicate with their memoryless offspring

[13] Ben-Porath and Kahneman (1996), Compte (1998), Kandori and Matsushima (1998) all examine communication in repeated games when players receive private signals of others' past behavior. Related ideas can also be found in Ahn and Suominen (2001).

who replace them in the game. AL examine the fragility of communication mechanisms when each individual is assumed to treat his successor's discounted utility as if it were his own.

Unlike that paper, the focus of the present paper is on certain types of organizations such as firms for which substantial altruistic links between managers of different cohorts within the same firm may not exist. Nevertheless, the present paper shows that a very small amount of altruism for one's successor can have large effects.

Finally, another vehicle for conveying information is a tangible asset such as money. A number of contributions in monetary theory have all shown, to varying degrees, the substitutability of money for memory.[14]

Several remarks about limitations and extensions are in order. First, the particular structure of communication in this model induce equilibria with "generation skipping" punishments. Punishment may not occur until after both the perpetrator and victim have "died."[15] It appears that this particular feature is, in some sense, canonical if communication is public. Other types of equilibrium sanctions probably exist as well. For example, strategies seemingly exist which induce natural correlation between messages and privately observed histories. In a two period OLG game, for instance, the young individual at date t may punish misreporting by his old counterpart by misreporting in the following period. This threat induces potential correlation between the original date t message and the history leading up to that message. Consequently, "cheap talk" is no longer cheap.

Second, the asynchronous timing of communication comes quite naturally here due to the overlapping nature of entry and exit. Because of this and the fact that messages are cheap talk, a sender of public information must always be indifferent between sending any of the messages that he might send in any continuation. But if for complexity reasons, the sender fails to distinguish states of the world for which he is indifferent, then communication will not be informative. In that case, the Folk Theorem fails.[16]

Third, the main result, the one with costly communication and altruism, assumes a single message sent publicly, from the outgoing member of an organization. Naturally, if the member of the other organization could simultaneously send a public message, then simple cross checking mechanisms exist to sustain any feasible payoff of the full memory game.[17]

[14] See, for example, Kocherlakota (1998), Kocherlakota and Wallace (1998), Wallace (2001), and Corbae, Temzelides, and Wright (2001).

[15] In many disputes, attacks directed from one group toward another are often justified as retaliatory punishments for transgressions that occurred long before anyone in the current cohort was around (e.g., conflicts in Northern Ireland, Bosnia, and the Middle East).

[16] A central result in Anderlini and Lagunoff (2001) for dynastic repeated games establishes that when communication is sequential and when payoffs incorporate the complexity of communication (even only lexicographically), then communication protocols sustain only the stage Nash equilibrium payoffs. In the present model, communication is subject to the same fragility.

[17] An explicit construction is given in Anderlini and Lagunoff (2001). Though the environment there is

If, on the other hand, the only message that can be used is private, and there is no public message available, then the punishment scheme we have described does not sustain cooperation. The problem arises when the old tries to send a message to his successor that he was cheated by the opponent's predecessor. In this case, he has to be indifferent between sending the cheapest message m^0 and the second cheapest one m^1. Since the cost of sending m^1 is greater than m^0, this implies that the old's successor should gain by receiving m^1 to offset the cost difference. Now, if this is the case, the old's successor, even if he receives m^0, has an incentive to pretend that he received m^1 instead because the opponent does not know which message was actually sent. Thus, the proposed profile does not constitute an equilibrium in case of private communication.

Fourth, the restriction to two organizations limits the applicability of results somewhat, although we are confident a modification of the model to finite numbers of organizations can be done. What makes the modification nontrivial is that it is not only the length of the lifetimes, but also the overlap between any two individuals' lifetimes that matter (see Kandori (1992)).

6 Appendix: Proof of the Main Result

6.1 Proof of Theorem 4

6.1.1 The Punishment Phases

The equilibrium will utilize two types of *punishment* phases. To describe these, first let $w^i = (v_i^*, w_j^i)$ satisfy $\varepsilon/2 < |w^i - v^*| < \varepsilon$ for each $i = 1, 2$, and

$$\begin{aligned} w_j^i > v_j^* \quad if \quad v_j^* > u_j(\underline{\sigma}^i) \\ w_j^i < v_j^* \quad if \quad v_j^* < u_j(\underline{\sigma}^i) \\ w_j^i \geq v_j^* \quad if \quad v_j^* = u_j(\underline{\sigma}^i) \end{aligned} \tag{4}$$

Roughly, j's component payoff of w^i dominates v^* iff, in turn, v^* dominates j's payoff when he minimaxes i. The feasibility of such a payoff profile w^i follows from the fact that $v^* \in int(V)$. The role of the vector w^i is to compensate the sanctioner, Player j (the individual who carries out the punishment), in the second phase of any punishment, while leaving the original perpetrator indifferent to the reference (equilibrium) payoff. Notice that if $w_j^i > v_j^*$ then the sanctioner is rewarded relative to the reference payoff. If $w_j^i < v_j^*$ then

slightly different, only minor modification is necessary in order to the sustain any feasible payoff, including those on the boundary, with simultaneous messages.

the sanctioner is penalized relative to the reference payoff. Notice that this happens only when $v_j^* < u_j(\underline{\sigma}^i)$, i.e., when "sanctioning is its own reward."

Now the punishment phases can be described:

I Standard Punishment (P^i-phase): this phase lasts P periods, and is invoked whenever one young individual deviates against another young individual. Set $P = P' + P''$. In particular, this punishment phase is decomposed into a minimax phase lasting P' periods, and a sanctioning-is-its-own-reward phase lasting P'' periods in which $w^i = (v_i^*, w_j^i)$ is received each period. The perpetrator $i(k)$ therefore receives a payoff of $P'\underline{u}_i + P''v_i^*$ in the P^i-phase, while the sanctioner receives $P'u_j(\underline{\sigma}^i) + P''w_j^i$. In other words, in this punishment phase, the sanctioner j minimaxes the perpetrator for P' periods in the first subphase, and then gets some reward lasting P'' periods in the second.

II Intergenerational Punishment (Q^i-phase): this phase, which differs in length for each organization, lasts $Q_i > P$ periods. This phase is invoked whenever a new entrant $j(k+1)$ and his rival in i receive a particular message, m^1, from the predecessor $j(k)$. As with standard punishment, set $Q_i = Q_i' + Q_i''$ so that the punishment phase is decomposed into a minimax phase lasting Q_i' periods, and a sanctioning-is-its-own-reward phase lasting Q_i'' periods. Hence, during this phase, the perpetrator $i(k)$ receives a payoff of $Q_i'\underline{u}_i + Q_i''v_i^*$, while the sanctioner from lineage j receives $Q_i'u_j(\underline{\sigma}^i) + Q_i''w_j^i$. As with the Standard Punishment (P^i-phase), in the Intergenerational Punishment Phase, the sanctioner j minimaxes the perpetrator in the first subphase, this time for Q_i' periods rather than for P' periods, then the sanctioner receives a Q_i'' period reward.

6.1.2 The Equilibrium Strategies

We now describe the equilibrium. Our equilibrium construction will depend on which case in (4) we are in. For now the construction will assume that "sanctioning hurts" case where $v_i^* > u_i(\underline{\sigma}^j)$. This is the case in which punishing another player is not too lucrative relative to the reference allocation. By (4), this entails that w_i^j be chosen so that $w_i^j > v_i^*$. The "sanctioning-is-its-own-reward" case in which $v_i^* \leq u_i(\underline{\sigma}^j)$ requires only a minor modification of the equilibrium which we will describe at the end.

Let σ^* attain v^*. Player i takes σ_i^* each period in equilibrium as long as no deviation has occurred. At the end of individual $i(k)$'s lifetime, he sends a message m^0 to his successor, $i(k+1)$ and the individual in the rival lineage, j. The message m^0 connotes "no deviation from one's rival." We normalize the cost so that $c(m^0) = 0$ (Alternatively, m^0 could connote the absence of communication). The other message m^j which might be used will connote a deviation having occurred in the rival lineage j. Clearly, $c(m^j) \equiv c > 0$.

Individuals in both lineages proceed according this stationary strategy unless a deviation occurs. At that point, the equilibrium can be described in terms of what happens after each of the three types of deviations we described above. To check these deviations, we utilize the Principle of Unimprovability whereby one need only check one-period deviations. To verify that the Principle does indeed hold in our environment, observe that our strategy profile has a recursive structure if the state space is taken to be a finite subset of the set $\{1, \ldots, T\} \times \{v : \ v \text{ is feasible}\}$.

(i) **Young cheats young** Let $i(k)$, $j(k')$ denote the two individuals with $i(k)$ deviating against $j(k')$. In this case, the deviation occurs at date t with $t \leq \min\{T_{i(k)}, T_{j(k')}\} - P$. After a deviation by $i(k)$, play proceeds to Standard Punishment (P^i-phase). If after the P^i-phase, if there is no further deviation, the players return to σ^* and payoff v^*. If both individuals are still young and player i subsequently deviates in the first subphase (the minimax phase), it is ignored. If $j(k')$ deviates, then the P^j-phase is initiated. If deviation occurs in the second subphase (the reward phase) and one of the players is still young, then the P^i-phase punishment (or P^j-phase) is restarted. If a subsequent deviation occurs when one of the players is old, then proceed to (ii) or (iii), depending on whether young cheats old or vice-versa.

(ii) **Young cheats old** Let $i(k)$ denote the young individual while $j(k')$ denotes the old. In this case, the old individual $j(k')$ has less than P periods left in his tenure. For the remainder of $j(k')$'s life, both individuals play some one-shot Nash equilibrium $\hat{\sigma}$ with payoff profile $\hat{v}$. At the end of old individual $j(k')$'s life, he sends message m^i to his successor, $j(k'+1)$, indicating that the young individual $i(k)$ deviated. Play then proceeds to the Intergenerational Punishment Phase (Q^i-phase). In this case, the punishment is applied to $i(k)$ by the successor $j(k'+1)$.

After the Q^i-phase, if there is no further deviation, the players return to σ^* and payoff v^*. As before, a subsequent deviation by the perpetrator during the minimax phase is ignored. Any other deviation proceeds to the beginning of (i).

(iii) **Old cheats young** Again, let $i(k)$ denote the young individual while $j(k')$ denotes the old. For the remainder of $j(k')$'s life, both individuals again play the one-shot Nash equilibrium $\hat{\sigma}$ with payoff profile $\hat{v}$. At the end of his life, old individual $j(k')$ sends message m^0 to his successor, $j(k'+1)$. Recall that m^0 indicates "no deviation" from one's rival (which is accurate since $j(k')$ was the one who deviated). At that point $i(k)$ and $j(k'+1)$ play σ^* giving payoff v^* each period until the end of $i(k)$'s lifetime. At the end of his lifetime, $i(k)$ send message m^j indicating an earlier deviation by lineage j (the message m^j need not distinguish whether the deviation came from $j(k'+1)$ or his predecessor, $j(k')$). Players $i(k+1)$ and $j(k'+1)$ then proceed to the Q^j-phase.

Figure 3 illustrates the transitions for each of the deviations described above, ignoring the delay (before the Q^i-phase) that occurs before punishment is initiated.

6.1.3 A Modification when Sanctioning is Own Reward

As for the "sanctioning-is-its-own-reward" case in which $v_i^* \le u_i(\underline{\sigma}^j)$, we need only a small modification in case (i) where young cheats young. This case is the same as before except:

If the deviation occurred while on the equilibrium path, then, after the P^i-phase, if there is no further deviation, then the players return to σ^* and payoff v^* just as before. If, however, the deviation by Player ℓ ($\ell = 1, 2$) occurred in the qth period of the Intergenerational Punishment (Q^i-phase), then, if there is no further deviation, the players then return to date $q + P^\ell$ in equilibrium continuation (either within the Q^i-phase or afterward) and play as if no P^ℓ-phase punishment had occurred.

Let $d_i = \max_{s,s'}(u_i(s) - u_i(s'))$, and $d = \max_i d_i$ so that d is the maximal, one shot gain in the stage game. For now we only assume $T/2 > Q_i + P$, $Q_i' > P'$ and $Q_i'' > P''$. More precise bounds on T, Q_i, and P will be given later. Except when explicitly stated otherwise, our constraints will apply both to the equilibrium constructions, i.e., the cases when $v_i^* > u_i(\underline{\sigma}^j)$, and when $v_i^* \le u_i(\underline{\sigma}^j)$.

We first identify incentive constraints for each of three types of deviations. Then we find bounds on $T, P', P'', Q_i', Q_i'', Q_j'$ and Q_j'' that satisfy these incentive constraints.

6.1.4 Young cheats young

Recall that we examine the case where $i(k)$ deviates against $j(k')$. To deter deviations, we requires that no young individual deviates against another. We must therefore check incentives both I along the equilibrium path, and in the P^i or Q^i phases.

First, to deter deviations along equilibrium path, it suffices that

$$d_i < Pv_i^* - [P'\underline{u}_i + P''v_i^*] \tag{5}$$

holds. In Inequality (5), the left hand side is the maximal gain from a one-shot deviation. The right side denotes the loss. Inequality (5) simplifies to

$$d_i < P'(v_i^* - \underline{u}_i) \tag{6}$$

Now check incentives for lineage i in punishment phase. Clearly, in either the P^i or Q^i-phases, we need not check incentives of the deviating individual in the minimaxing subphase

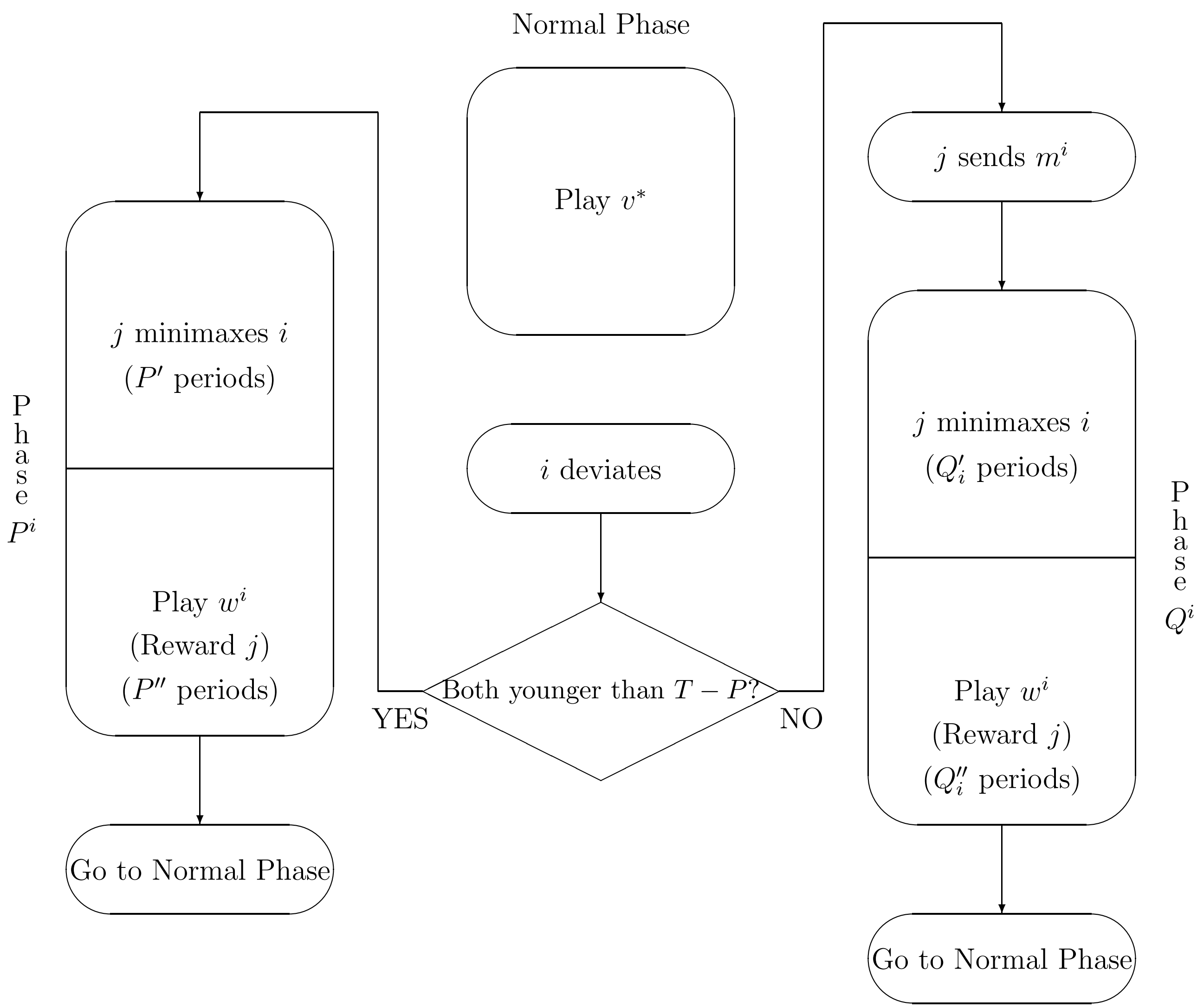

Figure 3: Basic Structure of Equilibrium Strategy Profile

since he already takes a stage best response. Moreover, in the P^i-phase, this simple penal code restarts if i deviates further. From the Q^i-phase, a deviation moves to the P^i phase if both individuals are young. As observed before, $i(k)$ will not deviate from his own Q'_i (minmax) phase. Since he already receives v_i^* in his own Q''_i-phase, he will not deviate from his Q''_i-phase if (6) holds.

However, for individual $j(k')$, his incentives depend on whether or not we in the "sanctioning hurts" case: $v_i^* > u_i(\underline{\sigma}^j)$.[18] We consider first the "sanctioning hurts" case, where $w_j^i > v_j^*$. Then a deviation in the P^i phase switches to the P^j phase. When $w_j^i > v_j^*$, it suffices to show that he will not deviate in the beginning of the P^i-phase:

$$d_j < [P' u_j(\underline{\sigma}^i) + P'' w_j^i] - [P' \underline{u}_j + P'' v_j^*] \tag{7}$$

or

$$d_j < P'(u_j(\underline{\sigma}^i) - \underline{u}_j) + P''(w_j^i - v_j^*) \tag{8}$$

As for a deviation from the Q^i-phase by j when $v_i^* > u_i(\underline{\sigma}^j)$, this initiates the P^j-phase, then play reverts to σ^* and v^*. Then, a sufficient deterrent for individual $j(k')$ is

$$d_j < [Q'_i u_j(\underline{\sigma}^i) + Q''_i w_j^i] - [P' \underline{u}_j + P'' v_j^* + (Q - P) v_j^*] \tag{9}$$

The right side of (9) is net loss from deviating by j at the beginning of the Q^i-phase. Since $w_j^i > v_j^*$ and $Q'_i > P'$ and $Q''_i > P''$, any deviations farther into the Q^i-phase increase the loss from deviating. To see this, we simplify (9) by rewriting it as

$$d_j < P'(v_j^* - \underline{u}_j) + Q'_i(u_j(\underline{\sigma}^i) - v_j^*) + Q''_i(w_j^i - v_j^*) \tag{10}$$

As one moves through the Q^i-phase, Q'_i is reduced, reducing the weight placed on $u_j(\underline{\sigma}^i) - v_j^*$ which in our "sanctioning hurts" case is negative.

Incentives when "Sanctioning is its Own Reward"

By contrast, in the "sanctioning-is-its-own-reward" case: $v_i^* \leq u_i(\underline{\sigma}^j)$, and so $w_j^i < v_j^*$ and a deviation from either the P^i-phase or the Q^i-phase by j initiates the P^j-phase, after which play reverts to the equilibrium continuation induced by the last message. In either the P^i or Q^i phases, individual $j(k')$ is most likely to deviate at the beginning of the second subphase of P^i or Q^i since he receives a payoff w_j^i below both v^* and $u_j(\underline{\sigma}^i)$. Hence, it is sufficient to deter deviation at the beginning of the second subphase of the P^i and Q^i phases. From the P^i-phase, the incentive constraint when $v_i^* \leq u_i(\underline{\sigma}^j)$ is:

$$d_j < [P'' w_j^i + (P - P'') v^*] - [P' \underline{u}_j + P'' v_j^*] \tag{7$'$}$$

where the right hand side is the net loss from deviating at the beginning of the P'' subphase. Inequality (7′) simplifies to

$$d_j < P'(v_j^* - \underline{u}_j) + P''(w_j^i - v_j^*) \tag{8$'$}$$

[18] This is the only place in the incentives where the modification of the equilibrium matters.

Starting from the Q_i'' subphase of Q^i:

$$d_j < Q_i'' w_j^i - [P' \underline{u}_j + P'' v_j^* + (Q_i'' - P) w_j^i] \tag{9'}$$

which we rewrite as

$$d_j < P'(w_j^i - \underline{u}_j) + P''(w_j^i - v_j^*) \tag{10'}$$

6.1.5 Young cheats old

Recall that this deviation occurs when old individual has P or fewer periods left to live. There are two incentives to check: (a) incentives for correct behavior and (b) incentives for correct communication.

(a) **Behavior** First, young individual $i(k)$ has no incentive to initially deviate against an old agent. The equilibrium path incentive constraint holds if

$$Pd_i < Q_i v_i^* - [Q_i' \underline{u}_i + Q_i'' v_i^*] \tag{11}$$

The left side of (11) denotes the maximal gain from deviating. In this case, when a young agent deviates against an old agent, he may gain no more than d_i for P periods since both the initial deviation and the one shot Nash equilibrium which follows lasts at most P periods until $j(k')$'s termination. The one-shot Nash play takes care of out of equilibrium incentives during young vs old interaction. Incentives against subsequent deviations are taken care of in Section 6.1.4 and the fact that both individuals $i(k)$ and $j(k')$ play the one shot equilibrium σ' until the end of $j(k')$'s lifetime. Rewriting (11) gives

$$Pd_i < Q_i'(v_i^* - \underline{u}_i) \tag{12}$$

(b) **Messages** The second incentive constraint entails that old agent $j(k')$ correctly send message m^i indicating a deviation by his rival. Since messages are cheap talk, this incentive constraint must be satisfied with equality since otherwise, m^0 would never be sent along the equilibrium path. This constraint is given by

$$\delta Q_i v_j^* = \delta[Q_i' u_j(\underline{\sigma}^i) + Q_i'' w_j^i] - c \tag{13}$$

The left side of (13) is the payoff to $j(k')$, lasting Q_i periods, from choosing equilibrium message m^0. Note that δ denotes the weight that $j(k')$ places on the payoffs of his successor individuals in the lineage. The cost c, however, is undiscounted. The right side is the punishment payoff during the Q^i-phase which is initiated when $j(k')$ chooses message m^i. (Note that since after Q_i periods, players revert to equilibrium play, the payoffs after Q_i periods on each side of equation (13) cancel out.)

6.1.6 Old cheats young

As with the previous case, there are two incentives to check: (a) incentives for correct behavior and (b) incentives for correct communication

(a) **Behavior** We must check that old individual $j(k')$ has no initial incentive to deviate. Note that, as before, subsequent incentives once a deviation occurs at this stage are taken care of first by the fact that the individuals play the one-shot Nash equilibrium σ' until the end of $j(k')$'s lifetime, and then by the previous cases.

Recall that in the description of the equilibrium, the deviating individual sends m^0 at the end of his life, and so punishment does not occur until the end of his rival's lifetime. Hence, the old individual's behavioral incentives are satisfied if

$$Pd_j < \delta\left\{(\frac{T}{2} + Q_j)v_j^* - [\frac{T}{2}v_j^* + Q_j'\underline{u}_j + Q_j''v_j^*]\right\}. \tag{14}$$

As before, the left hand side of (14) is the maximal gain from the initial deviation and subsequent play of one-shot Nash equilibrium, all of which lasts at most P periods. The right hand side is the loss. By deviating, individual $j(k')$'s successor $j(k'+1)$ obtains his equilibrium payoff v_j^* for a while — T/2 periods — until the end of $i(k)$'s lifetime. At this time $i(k)$ informs *his* successor that an earlier deviation occurred by sending m^j. Although $j(k'+1)$ was unaware of the earlier deviation, message m^j initiates the Q^j-phase, nonetheless. Rewriting (14) gives

$$Pd_j < \delta Q_j'(v_j^* - \underline{u}_j). \tag{15}$$

(b) **Messages** Finally, since $j(k')$ will only ever send m^0 in this case, his incentives need not be satisfied with equality as before. Since the Q^j-phase will begin once the young individuals $i(k)$'s life ends, regardless of j's message, $j(k')$ will correctly send m^0 if

$$\delta v_j^*\frac{T}{2} \geq \delta[Q_i'u_j(\underline{\sigma}^i) + Q_i''w_j^i + (\frac{T}{2} - Q_i)v_j^*] - c. \tag{16}$$

Note that since $j(k)$ knows that future messages will be conditioned on past behavior, his message does not solely determine the future play path.[19] Consequently, Inequality (16) need not hold with equality. Inequality (16) may therefore be rewritten as

$$\delta Q_i v_j^* \geq \delta[Q_i'u_j(\underline{\sigma}^i) + Q_i''w_j^i] - c \tag{17}$$

which is the weak inequality analogue of, and is therefore implied by, the equality constraint (13).

[19] Oddly, though future messages depend on information prior to the most recent message, our equilibrium remains a Perfect Public equilibrium since future behavior depends only on the most recent message.

6.1.7 Bounds on P, Q, and T

The relevant incentive constraints are (6), (8), (10), (12), (13), (15), and for the "Sanctioning-is-its-own-reward" case replace (8) and (10), with (8′), and (10′).

In either construction, set $Q'_i > P \cdot P'$. Then set $P' > \frac{d}{\delta(v_i^* - \underline{u}_i)}$ for each i. Together, these inequalities imply (6), (12), and (15).

Bounds for "Sanctioning Hurts" Case Consider first the construction for the "sanctioning hurts" case where $v_i^* \geq u_i(\underline{\sigma}^j)$. Then (10) is implied by (6) and (13). Now choose P'' so that

$$P'' > \frac{d}{w_i^j - v_i^*} + P' \frac{\underline{u}_i - u_i(\underline{\sigma}^j)}{w_i^j - v_i^*}, \ \forall i, \tag{18}$$

implying (8). Now having chosen bounds on P', P'', and Q'_i, choose Q''_i to satisfy

$$Q'_i(u_j(\underline{\sigma}^i) - v_j^* \ ) + Q''_i(w_j^i - v_j^*) = c/\delta, \tag{19}$$

which is equivalent to (13). By varying Q''_i and by varying the choices of both w^i and w^j in a way that maintains (4) and (18) above, the equality (19) can be satisfied given c and δ.

Bounds for "Sanctioning-is-its-own-reward" Case For this case, replace the bound on P'' in (18) above with

$$P' > \frac{d}{w_i^j - \underline{u}_i} + P'' \frac{v_i^* - w_i^j}{w_i^j - \underline{u}_i}, \ \forall i. \tag{20}$$

It implies both (8′) and (10′) given that $v_i^* > w_i^j$ holds in the "sanctioning-is-its-own-reward" case.

References

[1] Ahn, I. and M. Suominen (2001), "Word-of-Mouth Communication and Community Enforcement," *International Economic Review* 42: 399-415.

[2] Anderlini, L. and R. Lagunoff (2001), "Communication in Dynastic Repeated Games: 'Whitewashes and Coverups', " mimeo, Georgetown University.

[3] Aoyagi, M (2001), "Collusion Through Mediated Communication in Repeated Games with Imperfect Private Monitoring," mimeo.

[4] Bhaskar, V. (1998), "Informational Constraints and the Overlapping Generations Model: Folk and Anti-Folk Theorems," *Review of Economic Studies*, 65: 135-49.

[5] Benoit, and V. Krishna (1985), "Finitely Repeated Games," *Econometrica*, 53: 905-22.

[6] Ben-Porath, E. and M. Kahneman (1996), "Communication in Repeated Games with Private Monitoring," *Journal of Economic Theory*, 70: 281-97.

[7] Compte, O. (1998), "Communication in Repeated Games with Imperfect Private Monitoring," *Econometrica*, 66: 597-626.

[8] Corbae, D., T. Temzelides, and R. Wright (2001), "Endogenous Matching and Money," mimeo.

[9] Cremer, J. (1986), "Cooperation in Ongoing Organizations," *Quarterly Journal of Economics*, 101: 33-49.

[10] Hammond, P. (1975), "Charity: Altruism or Cooperative Egoism," in *Altruism, Morality and Economic Theory*, E. Phelps, Ed., New York: Russell Sage Foundation.

[11] Kandori, M. (1992) "Repeated Games Played by Overlapping Generations of Players," *Review of Economic Studies*, 59: 81-92.

[12] Kandori, M. and H. Matsushima (1998), "Private Observation, Communication, and Collusion," *Econometrica*, 66: 627-52.

[13] Kobayashi, H. (2000), "Folk Theorem for Infinitely Repeated Games Played by Organizations with Short Lived Members," mimeo, Kobe University.

[14] Kocherlakota, N. (1998), "Money is Memory," *Journal of Economic Theory*, 81: 232-51.

[15] Kocherlakota, N. and N. Wallace (1998), "Incomplete Record Keeping and Optimal Payout Arrangements," *Journal of Economic Theory*, 81: 272-89.

[16] Lagunoff, R. and A. Matsui (1997), "Asynchronous Choice in Repeated Coordination Games," *Econometrica*, 65: 1467-78.

[17] Prescott, E. and V. Rios-Rull (2000), "On the Equilibrium Concept for Overlapping Generations Environments," Federal Reserve Bank of Minneapolis Staff Report #282.

[18] Salant, D. (1991), "A Repeated Game with Finitely Lived Overlapping Generations of Players," *Games and Economic Behavior*, 3: 244-59.

[19] Samuelson, P. (1958), "An Exact Consumption Loan Model of Interest with or without Social Contrivance of Money," *Journal of Political Economy*, 66: 467-82.

[20] Shepsle, K. (1999), "From Generation to Generation: A Formal Analysis of Intertemporal Strategic Interaction," mimeo, Harvard University.

[21] Smith, L. (1992) "Folk Theorems in Overlapping Generations Games," *Games and Economic Behavior*, 4:426-49.

[22] Wallace, N. (2001), "Whither Monetary Economics," mimeo.

[23] van Damme, E. (1989), "Renegotiation in Repeated Prisoners' Dilemma," *Journal of Economic Theory*, 47, 206-17.