When Trade Requires Coordination*

Kimberly Katz Akihiko Matsui

February 2001 revised February 2003

JEL Nos.: C70, D62, F02, Z10

KEYWORDS: Assimilation, Behavioral Standards, (Perturbed) Best Response Dynamic, Coordination Costs, Gains from Trade, Imitation Dynamic, Integration, Monotone System, Replicator Dynamic.

Abstract

We consider a random matching model in which agents exchange endowments. Agents choose a standard of behavior from a given set of standards, with the knowledge that the gains from trade are higher if two trading agents have chosen the same standard. Agents have different preferences over these standards. There are two communities with uneven sizes. In one community, a particular standard is preferred by its members, while in the other, another standard is preferred. Using this framework, we study conditions under which, when we account for these inherent preferences over a set of standards, total welfare of a minority community decreases when a trade barrier between the two communities is lifted.

Corresponding author:

Akihiko Matsui
Faculty of Economics, University of Tokyo
Hongo, Bunkyo-ku, Tokyo 113-0033, JAPAN

e-mail: amatsui@e.u-tokyo.ac.jp

Website URL: http://www.e.u-tokyo.ac.jp/~amatsui/
(The .pdf file of the current version of the paper is available at this web site.)

*We thank two anonymous referees and seminar participants at the University of Pennsylvania as well as Stephen Coate, Ed Green, Atsushi Kajii, Mitch Kaneda, Stephen Morris and Andrew Postlewaite for comments and references. The second author gratefully acknowledges financial support from Grants-in-aid of the Ministry of Education and Sciences of Japan.

1 Introduction

Interaction requires coordination, and the interactions which arise in economic activities are certainly no exception. International trade, for example, requires coordination on a variety of conventions including the language used in negotiations. Corporate mergers often require that the merging companies coordinate on a single corporate culture. Network externalities force agents to coordinate on one or, at most, a small number of operating systems. Rural to urban migration results when people must coordinate on a geographical location for many nonagriculture sector jobs. Less obvious, but nonetheless important, examples can be found in behavioral assimilation of minority groups into majority groups. Immigrants often assimilate into the predominant culture of their new country, or phrased differently, they coordinate their customs with those of their new compatriots. And members of racial or ethnic groups which have historically interacted largely with only members from their own community find that a similar predicament applies to them. This paper studies the effects of integration of two communities in the presence of such need for coordination. It provides us with a result, *albeit* in a limited situation, against the received wisdom that trade enhancing integration *always* increases the welfare of the two communities.[1] In the sense that we do not present general and comprehensive results, the purpose of this paper is suggestive rather than definitive in nature.

If two communities are socially and/or economically integrated, members of a minority group may have the incentive to adopt behaviors of a majority group even if they inherently prefer their own behavior patterns, since adopting these alternative behaviors allows them to interact with a larger group of people. This force for coordination is referred to as strategic complementarity, and its effects have been studied extensively in development economics, international economics, and game theory.[2] Yet, welfare analysis is somewhat limited. Only gains from trade and coordination are often mentioned, and focuses are on how coordination on a desirable outcome can be achieved through various measures.[3]

The observation which we incorporate into the present model is the fact that different agents may have different inherent preferences over a set of standards on which they coordinate.[4] For instance, speaking a foreign language requires an additional effort compared to using one's mother tongue. The expenses of moving one's home from one location to another consist not only of direct moving expenses but also of various adjustment costs. And changing one's behavior patterns to those which are less natural or instinctive can be

[1]The only exceptions that we know of are the economies with incomplete markets (Hart; 1975) and the arguments related to custom unions (see, e.g., Baldwin and Venables; 1995).

[2]The literature on strategic complementarity is extensive. See Ray (1998) for arguments and references in development economics and international economics, and Kandori (1997) for those in game theory.

[3]One exception that we know of is Akerlof and Kranton (2000), which we will mention later in the introduction again.

[4]Akerlof and Kranton (2000) call such idiosyncratic preferences identities.

uncomfortable or even traumatic, as is eloquently phrased by Isaac Asimov (*Yours, Isaac Asimov*):[5]

> I am the son of immigrant parents. ... I had to learn American culture on my own, and it is difficult to explain to someone who has not gone through it what this means. ... In short, I was a cultural orphan (like many others) and "speak the culture with an accent."

Incorporating such heterogeneity in inherent preferences over standards of behavior, we study the relationship between the need for coordination and the standards of behavior people choose. In particular, we explicitly study a dynamic process and some welfare implications. In so doing, we find that when a small community is integrated with a large community, the small community ends up worse off than before provided that standard gains from trade based on comparative advantage are sufficiently small.

Roughly speaking, we construct a random matching model of trade with the following characteristics. First, gains from trade are higher if two agents use the same standard of behavior. Each agent chooses such a standard from an exogenously given set of standards before he/she is randomly matched with a trading partner. If a matched pair of agents have chosen the same standard, and if a beneficial trade exists, then they will trade their goods. If they have not chosen the same standard, then trade may be possible, but agents incur a significant cost due to the lack of coordination. We normalize this payoff to zero in the model.[6]

Second, all members of society belong to one of two distinct communities with different sizes, with members of one community preferring one standard while members of the other community prefer another. The costs of adopting a less preferred standard may differ among agents within a community.

We focus our analysis on the situations in which one country is smaller than the other in terms of the production of the two goods, so that the former is unambiguously smaller than the latter. This assumption, albeit not universal, is not unrealistic. One may think of situations such as trades between the United States and Kuwait where the United States produces far more crude oil than Kuwait (US, 319Mt; Kuwait, 100Mt, 1996; United Nations Monthly Bulletin of Statistics 2001), but the former imports it, while the latter exports it (US's import, 40 billion dollars; Kuwait's export, 14 billion dollars, 1996; ibid).

As a reference point, we begin with a situation in which agents trade only with members of their own community. This corresponds to situations in which there is a barrier between

[5] Other references include Rubin (1995), Cose (1993), and de Beauvoir (1952) among others. The secession movement of Quebec in 1990's provides us with another example. According to The Gazette (Montreal; January 20, 1994, Thursday, FINAL EDITION), Bouchard, an ex-premier of Quebec, 'said Quebecers will always be in a minority in Canada. "This handicap makes Quebecers second-class citizens," he said.'

[6] This normalization certainly loses some generality since it implies that trading using different standards is equivalent to no trading. However, there will be little qualitative change in the results.

the two communities, in the sense that a member of one community simply does not come into contact with members of the other community. We assume that each community is in the equilibrium in which all members choose the standard preferred within that community. We then study a dynamic process and its long-run result after this barrier is lifted. It is shown that there is a unique equilibrium which is accessible from the initial autarkic situation under any monotone dynamic, and that in the process of moving toward the new equilibrium, members of the smaller community assimilate into the larger community.[7]

We make welfare analysis, comparing autarky and free trade, which is made possible by the uniqueness of the accessible outcome after the barrier is lifted. In the main part of welfare analysis, we assume that the matching technology exhibits constant returns to scale. In this case, there are essentially two effects of integration. If there is no inherently preferred standard, then integration increases the welfare of the both communities due to standard gains from trade based on uneven distributions of endowments. This is the positive effect of integration. On the other hand, due to distinct inherent preferences over a set of standards, those who assimilate into the larger community incur costs, which induces a negative effect. The total effect of integration is determined by the relative size of these two effects. If gains from trade are minimal, then the loss due to adopting less preferred standard becomes a dominant factor, and total welfare decreases when the barrier is lifted.

We also analyze the case in which the matching technology exhibits increasing returns to scale. In this case, integration increases the matching probability for each agent. This economy of scale induces a positive effect on welfare when the communities are integrated. Therefore, nonexistence of gains from trade based on comparative advantage does not provide a sufficient condition for a decrease in welfare after integration. Still, there are nontrivial situations in which integration leads to a welfare loss.

Under the assumption of increasing-returns-to-scale matching technology, we also look briefly at a situation in which agents may choose from a set of more than two standards. We offer an example in which, beginning from the autarkic situation in which all agents use their respective most preferred standards, all agents in both communities change their standard to a third standard when the barrier between the communities is lifted. This third standard is not the most preferred standard of either community but is most "easily accessible" by both. We identify some conditions under which all members in both communities are left worse off.

The present paper is built on the framework constructed by Matsui and Okuno-Fujiwara (2002), who consider a situation in which players from two regions are matched to play some coordination games. They study conditions under which complete and partial assimilation

[7]The class of dynamics we consider include the replicator dynamic (Taylor and Jonker; 1978), the (perturbed) best response dynamic (Gilboa and Matsui; 1991), and imitation dynamic (Schlag; 1998) as special examples. The assumptions imposed upon the dynamics are fairly mild and satisfied by virtually all the standard dynamics used in the literature on evolutionary game theory and search theory. See also Hofbauer and Sandholm (2002) for some of such dynamics.

occur as a result of integration of the two regions. They also show that eclectic (hybrid) behavior may arise as a result of integration.

The present paper is also related to Lazear (1999). He examines which minority group tends to learn English faster than others after migration to the United States. He finds, among others, that the larger the size of the group is, the slower people in the group learn it. Members of a smaller group may find it urgent to coordinate with the majority of the society than those of a larger group. Strategic complementarity is the main source to account for this phenomenon. The present paper goes beyond these works to claim that as a result of such assimilation, the minority group may be worse off.

Another paper closely related to the present paper is Akerlof and Kranton (2000). Different inherent preferences over a set of standards can be interpreted as "identities" in their terminology. They compare several equilibria when two groups of people interact with each other. The main difference between their paper and ours lies in the fact that we identify a single equilibrium which is accessible from the autarky equilibrium, and thereby make it possible to compare welfare before and after a trade barrier is lifted.

The logic we use in the paper to show that one community may be worse off when the barrier is lifted differs from that found in Hart (1975) and in the literature on customs unions,[8] both of which also examine cases in which a welfare loss can result upon the lifting of a trade barrier. First, Hart examines a case involving incomplete markets and shows that the addition of an asset which allows trade between some of the markets, but not all of them, can lead to a decrease in welfare. Second, in the literature on customs unions, it is shown that when a country has two trading partners, one with high costs and the other with low costs, and this country forms a customs union with the higher cost partner, while imposing a high tariff on the low cost partner, then a decrease in welfare is also possible. Both examples essentially claim that a "partial shift" toward the first best does not necessarily improve welfare. Instead of using this logic, our welfare result relies on the negative externality induced by a switch from a favorable standard to an unfavorable one.

The remainder of the paper is organized as follows. The formal model is laid out in Section 2. Section 3 characterizes equilibria, which is followed by the analysis of dynamics in Section 4. An analysis of the welfare implications which result from our accounting for the cost of coordination makes up Section 5. Section 6 assumes that the matching technology exhibits increasing returns to scale, and continues our welfare analysis. Although the welfare implications are not as clear-cut as in the case of constant returns to scale, two examples are offered to show a decrease in welfare after integration. Section 7 concludes the paper.

[8] See Baldwin and Venables (1995) for discussions and references.

2 The Model

We consider an exchange economy consisting of two types of agents, type 1 and type 2. The size of the type 1 group and that of the type 2 group are equal. There are two indivisible commodities, 1 and 2. A type 1 agent is endowed with 2 units of good 1, while a type 2 agent is endowed with 2 units of good 2. Each type 1 agent is indexed by a number in $(0, 1)$, as is each type 2 agent. Agents get zero utility from consuming only one good, while they get positive utility from consuming a unit each of 1 and 2. In this world, trade takes place in the form of a one-for-one swap of goods. Agents cannot trade their goods unless they coordinate on one of two behavioral standards, L and R.

Each agent belongs to one of two communities, A and B. Agents in A prefer coordination on standard L, while agents in B prefer coordination on R. However, coordination on some standard is preferred to miscoordination by all agents. If an agent i of type 1 (resp. 2), from community A, uses standard L and trades with an agent of the opposite type using L, the agent receives a utility level which we normalize to 1, while if he uses standard R and trades with another agent using R, he receives utility $\mu_R(i) \in [0, 1]$ (resp. $\nu_R(i) \in [0, 1]$). Similarly, if an agent j of type 1 (resp. 2) from community B uses standard R and trades with an agent of the opposite type using R, the agent receives utility 1, while if he uses standard L and trades with another agent using L, he receives utility $\mu_L(j) \in [0, 1]$ (resp. $\nu_L(j) \in [0, 1]$).[9]

We denote the fraction of the type 1 agents in community A by m and the fraction of the type 2 agents in community A by n. Thus, the fraction of the total population of community A is $\frac{m+n}{2}$. We arrange type 1 agents uniformly on the line [0,1] in the following manner: for $i \leq m$, $0 \leq \mu_R(i) \leq 1$ and $\mu_L(i) = 1$; for $i > m$, $0 \leq \mu_L(i) \leq 1$ and $\mu_R(i) = 1$. In addition, $\mu_R(i)$ is (weakly) increasing, and $\mu_L(i)$ is (weakly) decreasing. This places players in community A on the first portion of the line and those in B after them. Also, note that this is equivalent to placing the players who have the most trouble changing their standard of behavior at the outer ends of the line, while placing those who would find it least difficult to make this change nearest the members of the community to which they do not belong. In the same manner, we arrange type 2 players on the line [0,1] as follows: for $j \leq n$, $0 \leq \nu_R(j) \leq 1$ and $\nu_R(j) = 1$; for $j > n$, $0 \leq \nu_L(j) \leq 1$ and $\nu_L(j) = 1$; $\nu_R(j)$ is increasing, and $\nu_L(j)$ is decreasing.

There is no centralized market where agents can meet to exchange commodities. Rather, agents are randomly matched into pairs. We examine two cases. We will refer to the first of these cases as the "autarky" case, which will serve as a benchmark for the second case, or the "unification" case. In the autarky case, agents are matched only with agents from

[9]Note that by defining μ and ν as we do, our model can also be interpreted to address situations in which agents from one community can interact profitably with members of the other community, but they may not be able to fully reap the benefit of the interaction in the way that the agents using their most preferred standard can.

their own community. Matching is uniform within each community, and the matching technology exhibits constant returns to scale. In other words, a type 1 (resp. type 2) agent in community A meets a type 2 (resp. type 1) agent with probability $\frac{n}{m+n}$ (resp. $\frac{m}{m+n}$). Similarly, a type 1 (resp. type 2) agent in community B meets a type 2 (resp. type 1) agent with probability $\frac{1-n}{2-m-n}$ (resp. $\frac{1-m}{2-m-n}$).

The second case is the "unification" case. Here, agents may meet trading partners from either community. Matching is uniform. In this case, an agent will meet a trading partner, i.e., an agent of the opposite type, with probability 1/2. Recall, however, that this is not equivalent to saying that an agent will trade with probability 1/2. Agents in a pair must use the same standard of behavior.

The fractions of agents who are matched with agents of the opposite type are $\frac{2mn}{(m+n)^2}$ in the autarky case, and 1/2 in the unification case, respectively. This implies that if $m \neq n$, i.e., if each community has a comparative advantage in one of the goods, then the fraction of matches between two distinct types of agents is greater in the unification case than in the autarky case. Gains from trade would arise if it were not for the need for coordination.

3 Equilibria

3.1 Autarky

In the autarky case, since no agent can meet an agent from the other community, we can analyze each community separately. We consider community A. The analysis for community B is symmetric. First, any pure strategy equilibrium can be characterized by two numbers, $x \in [0, m]$ and $y \in [0, n]$, where agent i of type 1 (resp. type 2) takes standard L if and only if $i < x$ (resp. $i < y$).[10] Indeed, if $i < j$, then $\mu_R(i) \leq \mu_R(j)$ and $\nu_R(i) \leq \nu_R(j)$ and therefore, if player i takes R in an equilibrium, player j of the same type weakly prefers R to L as well.

On average, the agent i of type 1 obtains $\frac{y}{m+n}$ if he takes L and $\frac{n-y}{m+n}\mu_R(x)$ if he takes R. Therefore, his incentive conditions are given by

$$\begin{aligned} y &\geq (n-y)\mu_R(i) \qquad \text{if } i < x, \text{ and} \\ y &\leq (n-y)\mu_R(i) \qquad \text{if } i > x. \end{aligned}$$

These inequalities give the incentive curve for type 1 agents, i.e., the curve on which no agent of type 1 has an incentive to deviate:

$$y = (n-y)\mu_R(x). \tag{1}$$

[10] This is unique up to permutation among those with the same μ's and ν's. Also, we ignore the action taken by the agent at the thresholds x and y.

Similarly, for an agent i of type 2, we have

$$\begin{aligned} x &\geq (m-x)\nu_R(i) && \text{if } i < y, \text{ and} \\ x &\leq (m-x)\nu_R(i) && \text{if } i > y. \end{aligned}$$

These inequalities give the incentive curve for type 2 agents:

$$x = (m-x)\nu_R(y). \tag{2}$$

The intersections of (1) and (2) determine the equilibria of this community. Since $\mu_R(x)$ and $\nu_R(y)$ are functions of x and y, respectively, we can draw equilibrium conditions on a (x, y)-plane.

There are always two equilibria, $(0, 0)$ and (m, n) since there is no gain from trade by taking a standard that is taken by nobody.[11]

Two examples are given in Figures 1 and 2. The first illustrates the case in which μ and ν are distributed uniformly, i.e.,

$$\mu_R(i) = i/m \tag{3}$$

and

$$\nu_R(i) = i/n. \tag{4}$$

While the second illustrates the case where the distributions are made up of two mass points at 0 and 4/5 with a quarter of each type having 0, i.e.,

$$\mu_R(i) = \begin{cases} 0 & \text{if } i \leq \frac{1}{4}m, \\ \frac{4}{5} & \text{if } i > \frac{1}{4}m, \end{cases} \tag{5}$$

and similar for $\nu_R(\cdot)$. The arrows in the figures show their incentives.

In the second example, there are other equilibria than the two equilibria, $(0, 0)$ and (m, n). Outcome $(\frac{1}{4}m, \frac{1}{4}n)$ is one of them. In this equilibrium, those who obtain no utility from coordinating on R stick to standard L, while those who obtain a positive utility take standard R. The community is divided into two in this equilibrium.

3.2 Unification

The equilibrium analysis of the case with no barrier is similar to that of the autarky. As before, an equilibrium is now characterized by four thresholds, $x_A, x_B \in [0, m]$ and $y_A, y_B \in [m, 1]$. The type 1 agent at x obtains $y\mu_L(x)$ if he takes L, and $(1-y)\mu_R(x)$ if

[11]This result might change if there are positive gains from trade even if the partner takes a different standard, and if some agents really dislike one of the standards.

he takes R. Therefore, the incentive conditions for type 1 agents in both communities are given by

$$\begin{aligned} y\mu_L(i) &\geq (1-y)\mu_R(i) \qquad \text{if } i < x, \text{ and} \\ y\mu_L(i) &\leq (1-y)\mu_R(i) \qquad \text{if } i > x. \end{aligned}$$

Similarly, the incentive conditions for type 2 agents are given by

$$\begin{aligned} x\nu_L(i) &\geq (1-x)\nu_R(i) \qquad \text{if } i < y, \text{ and} \\ x\nu_L(i) &\leq (1-x)\nu_R(i) \qquad \text{if } i > y. \end{aligned}$$

Therefore,

$$y\mu_L(x) = (1-y)\mu_R(x) \tag{6}$$

and

$$x\nu_L(y) = (1-x)\nu_R(y) \tag{7}$$

jointly determine the incentive curves and hence equilibrium. For every set of parameters, there are at least two equilibria, (0,0) and (1,1). These equilibria are called *completely assimilated equilibria*. In these equilibria, all members of one community have coordinated with, or assimilated into, the other community. In general, there are more than two equilibria. An equilibrium (x^*, y^*) is said to be a *partially assimilated equilibrium* if

$$(x^*, y^*) \in [0, m] \times [0, n] \cup [m, 1] \times [n, 1] \setminus \{(0,0), (m, n), (1,1)\}.$$

Here, some members of one community have coordinated with the members of the other community, but not all members have done so. One community is divided due to integration with the other community.

Figures 3 and 4 illustrate two examples (ignore arrows for the moment). Figure 3 corresponds to the case of uniform distribution given by (3) and (4) for community A, and symmetrically,

$$\mu_L(i) = \frac{1-i}{1-m}, \; i > m, \tag{8}$$

and

$$\nu_L(i) = \frac{1-i}{1-n}, \; i > n, \tag{9}$$

for community B. In the figure, we let $m = 0.4$ and $n = 0.25$ for the sake of illustration. Recall that $\mu_L(i) = 1$ holds for $i \leq m$, and so on.

Figure 4 corresponds to the case where there are two mass points in each community in terms of the distributions of μ's and ν's. There is a seemingly stable partially assimilated equilibrium, but in order to see the stability of equilibria and which equilibrium may arise as the result of lifting the barrier, we must turn to dynamical analysis.

4 Dynamics

We study dynamics after the barrier is lifted. To this aim, we assume that time is continuous, and the horizon is infinite. Each agent of type 1 (resp. 2) can produce two units of good 1 (resp. 2) at a time right after he/she consumes a unit each of the two goods.[12]

In general, we do not know which equilibrium emerges as the result of integration if there are multiple equilibria. Indeed, it is the nature of conventions/standards that the outcome is history dependent. If the two communities coordinate on the same standard at the time of integration, then this standard continues to be used after the integration, and there would be no issue of assimilation.

Therefore, we analyze below a more interesting case than this, i.e., the situation in which different communities adopt different standards. In particular, we focus our attention on the situation in which agents in each community coordinate on their respective preferred standards before the barrier is removed. It corresponds to the case in which (m, n) is the initial condition.

The first subsection, which is the main part of the present section, studies situations in which community A is smaller than community B in both types. The second subsection briefly looks at situations in which community B contains more type 1 agents, but less type 2 agents, than community A.

4.1 Small Community vs. Large Community

Throughout this subsection, we assume $m < 1/2$ and $n < 1/2$, i.e., community A is smaller in both types than community B. We use a class of dynamical processes to select equilibrium, which generically makes our results conclusive if the initial condition is (m, n). In the class of dynamical processes, agents gradually adjust their behavior. Such a slow adjustment process is appropriate in our problem, since cultural traits change only slowly. We assume that time is continuous, as is the dynamical path. For the sake of simplicity of the analysis, we assume further that if many agents have incentives to switch their actions, those who have greater incentives than others switch first. This assumption enables us to characterize the state of the dynamical system by two thresholds x and y on condition that the initial condition is also expressed by two thresholds as we have assumed.[13] We assume that $\mu_R(\cdot)$, $\nu_R(\cdot)$, $\mu_L(\cdot)$, and $\nu_L(\cdot)$ are all Lipschitz continuous, i.e., for all i, there exist

[12] Another mathematically equivalent assumption is that the agent who consume leaves the market and is replaced by a new agent with some endowment. Either of these assumptions makes the system time-independent and thereby simplifies the analysis. For this reason it is adopted as a standard formulation in the literature of micro-foundation of market transactions (e.g., Gale (1986) and Rubinstein and Wolinsky (1985)) and search theoretic models of money (e.g., Kiyotaki and Wright (1989) and Matsuyama, Kiyotaki, and Matsui (1993)).

[13] Note that the initial condition is not expressed by two thresholds if both L and R are taken in each community.

$\varepsilon > 0$ and $K > 0$ such that for all $j \in (i-\varepsilon, i+\varepsilon)$, $|\mu_R(i) - \mu_R(j)| < K|i-j|$ holds, and so forth.

Formally, consider the following system of differential equations:

$$\begin{cases} \dot{X} &= F(X,Y) \equiv f\left(X, \mu_L(X)Y - \mu_R(X)(1-Y)\right), \\ \dot{Y} &= G(X,Y) \equiv g\left(Y, \nu_L(Y)X - \nu_R(Y)(1-X)\right), \end{cases} \tag{10}$$

where f and g are Lipschitz continuous, continuously differentiable, (weakly) increasing in the second argument, and satisfy the following:

$$\begin{aligned} f(X,0) &= g(Y,0) = 0, \\ f(X,Z) &> 0 \quad \text{if } Z > 0,\ X \neq 1, \\ f(1,Z) &= 0 \quad \text{if } Z > 0, \\ f(X,Z) &< 0 \quad \text{if } Z < 0,\ X \neq 0, \\ f(0,Z) &= 0 \quad \text{if } Z < 0, \\ g(Y,Z) &> 0 \quad \text{if } Z > 0,\ Y \neq 1, \\ g(1,Z) &= 0 \quad \text{if } Z > 0, \\ g(Y,Z) &< 0 \quad \text{if } Z < 0,\ Y \neq 0, \\ g(0,Z) &= 0 \quad \text{if } Z < 0. \end{aligned}$$

In (10), $\mu_L(X)Y - \mu_R(X)(1-Y)$ (resp. $\nu_L(Y)X - \nu_R(Y)(1-X)$) is the payoff difference between L and R for the type 1 (resp. 2) agent at the threshold X (resp. Y). It is verified that every limit point is an equilibrium; otherwise, the solution path moves away from it. This dynamic process enables us to draw a phase diagram. Two examples of phase diagrams are shown in Figures 3 and 4 (see the previous subsection for the explanation of these figures).

Since F and G are Lipschitz continuous, there is a unique solution to the system in $[0,1]^2$ if the initial condition is given. Note that this is a canonical dynamic process. All we need is that the system moves in a (Lipschitz) continuous manner until there is no person who can be better off by switching his standard.

In order to characterize the limiting behavior of the dynamic with the initial state (m,n), let

$$x^* = \max_{x<m}\{(x,y) | F(x,y) = G(x,y) = 0\}, \tag{11}$$

and

$$y^* = \max_{y<n}\{(x,y) | F(x,y) = G(x,y) = 0\}. \tag{12}$$

Note that $F(0,0) = G(0,0) = 0$ holds, while $F(m,n) = G(m,n) = 0$ does not hold, and therefore, both x^* and y^* exist. Moreover, since the loci of $F(x,y) = 0$ and $G(x,y) = 0$ are upward sloping, x^* and y^* are uniquely defined, and $F(x^*, y^*) = G(x^*, y^*) = 0$ holds.

Next, it follows from the next lemma found in Smith (1988) that this system is a *monotone system*, i.e., for all initial conditions z_0 and w_0 with $z_0 \geq w_0$ implies $z(t) \geq w(t)$ for all $t > 0$ since $\partial F/\partial Y = f_2[\mu_L(X)+\mu_R(X)] \geq 0$ and $\partial G/\partial X = g_2[\nu_L(Y)+\nu_R(Y)] \geq> 0$ where f_2 (resp. g_2) is the partial derivative of f (resp. g) with respect to the second argument.

Lemma 1 *[Smith (1988, p.91)] The system given by (10) is a monotone system if and only if the off-diagonal elements of the Jacobian of (10) are nonnegative, i.e., $\partial F/\partial Y \geq 0$ and $\partial G/\partial X \geq 0$.*

It also follows from another lemma found in Smith (1988) that in a bounded monotone system, the solution converges to an equilibrium point.

Lemma 2 *[Smith (1988, p.94)] Consider a solution $(X(\cdot), Y(\cdot))$ in a bounded monotone system. If $X(\tau) < X(0)$ and $Y(\tau) < Y(0)$ hold for some $\tau > 0$, then $(X(t), Y(t))$ converges to an equilibrium as t goes to infinity.*

We now have the following proposition.

Proposition 1 *The unique equilibrium that is accessible from (m,n) is (x^*, y^*) given by (11) and (12).*

Proof. Consider $K = [x^*, m] \times [y^*, n]$. Since the present system is monotone, $F(x^*, y^*) = G(x^*, y^*) = 0$, $F(m,n) < 0$, and $G(m,n) < 0$ imply $F(x^*, \cdot) \geq 0$, $F(m, \cdot) \leq 0$, $G(\cdot, y^*) \geq 0$, and $G(\cdot, n) \leq 0$ on K. Therefore, the restriction of (10) on K is a self-contained system, i.e., the path would never go out of K so that we can consider it as a bounded monotone system on K.

Suppose that (m,n) is the initial condition. Since $F(m,n) < 0$ and $G(m,n) < 0$ hold, there exists $\tau > 0$ such that $X(\tau) < m$ and $Y(\tau) < n$ hold. Thus, applying Lemma 2, we establish that the system converges to an equilibrium. By definition, the only equilibrium in K is (x^*, y^*). Hence, the system converges to (x^*, y^*). ∎

An immediate corollary of this proposition is that it is always the case that upon lifting the barrier, members of the smaller community assimilate into the larger community. Also, note that the equilibrium accessible from (m,n) does not depend upon the relative speed of adjustment, α and β. Whether the system reaches a completely assimilated equilibrium or a partially assimilated one depends upon μ_R and ν_R. In Figure 3, completely assimilated equilibrium (0,0) is accessible from (m, n). On the other hand, in Figure 4, the uniquely accessible equilibrium from (m, n) is the partially assimilated equilibrium (x^*, y^*).[14]

Proposition 1 enables us to do some comparative statics. We compare two societies in terms of the degree of assimilation. Consider two societies in both of which (m, n) represents community A as well as the initial point. One society has distributions μ and ν, and the other has μ' and ν'. Let (x, y) be the equilibrium accessible from (m, n) in the first society, and x', y' be the equilibrium accessible from (m, n) in the second society. If $\mu_R(i) < \mu'_R(i)$ holds for all $i \in [0, m)$ and $\nu_R(i) < \nu'_R(i)$ holds for all $i \in [0, n)$, then we have

$$x \geq x', \text{ and } y \geq y',$$

where strict inequalities hold whenever x' and y' are strictly positive. In other words, if agents in community A of one society dislike R more than those in community A of the other society, then the number of people assimilating into community B in the first society is no more than that in the second society. To see this point, note that F shifts upward and G shifts downward, which makes the intersection close to the origin.

4.2 Incomparable Community Sizes

If neither community is larger than the other in both types of agents, *e.g.*, if $m < 1/2 < n$, there may be different equilibria accessible from (m, n) depending upon the relative speed of adjustment between types 1 and 2. Figure 5 shows such a possibility. In this case, we cannot determine which community will gain more after the barrier is lifted. If, for example, the speed of adjustment of community A is sufficiently slower than that of community B, then it is community A that benefit more from free trade since agents in community B adjust to L before those in A adjust to R.

4.3 Different Initial Conditions

If we take a different distribution under autarky as the initial condition, then the result would change. The easiest and trivial cases are the ones in which the two communities adopt the same standard from the beginning. These cases are reduced to the ones without multiple standards, and no issue of assimilation would arise.

Another less obvious and potentially interesting case is when agents of each community coordinate upon their less preferred standard, i.e., standard R is taken in community A, while standard L is taken in community B. In this case, we cannot use the technique

[14] Note that the second example does not satisfy Lipschitz continuity in $[0, 1]^2$. This is for the sake of illustration. It is verified, however, that in this particular example, one can show that there is a unique solution path exists, and that it converges to (x^*, y^*). Indeed, in the region containing (m, n), both x and y decreases. Once the system hits either one of the two incentive curves, it moves straight along the incentive curve it hit until it reaches (x^*, y^*).

developed in the main part of the analysis since $i < j$ no longer implies that agent i takes R only if agent j takes R. Indeed, the system becomes four dimensional, and without some simplification, we cannot analyze its general properties. One simplification is to make the two types symmetric, so that the dynamic of type 1 agents is the same as that of type 2 agents, which we assume throughout this subsection.

For this purpose, we assume that $m = n$ holds, and that $\mu_R(i) = \nu_R(i)$ and $\mu_L(i) = \nu_L(i)$ hold for all $i \in (0, 1]$.[15] Moreover, the adjustment speed is assumed to be the same between types 1 and 2. We still assume, however, that in each community, agent i takes R only if agent j takes R for all i and j with $i < j$. This assumption enables us to characterize the system by two thresholds, z_A and z_B, where for $k = A, B$, $i < z_k$ (resp. $i > z_k$) implies that agent i of both types of community k takes standard L (resp. R). The system is bounded by $[0, m] \times [m, 1]$. The initial condition $(z_A(0), z_B(0))$ is equal to $(0, 1)$.

5 Welfare Implications

The previous section has established that there is the unique equilibrium that is accessble from the autarky equilibrium (m, n) provided that $m, n < 1/2$ holds, which we assume throughout this section. This enables us to turn to the welfare implications of our accounting for the inherent preferences over a set of standards when the two communities are integrated. In the sequel, therefore, we focus on the equilibrium which is accessible from the initial condition (m, n).

We use a simple welfare function in which the utility of each member in the community is given equal weight. Thus, we define total welfare W for community A to be the sum of the total welfare of A's type 1 agents, W_1, and the total welfare of the type 2 agents, W_2. If a pair of threshold (x, y) satisfies $x < m$ and $y < n$ as in the accessible equilibrium we focus on, they are calculated as follows:

$$W_1 = \frac{xy}{2} + \int_x^m \mu_R(i) \frac{1-y}{2} \, di, \tag{13}$$

and

$$W_2 = \frac{xy}{2} + \int_y^n \mu_R(i) \frac{1-x}{2} \, di. \tag{14}$$

where, for example, the first term of (13) is the sum of expected payoffs of non-assimilating agents of type 1, and the second term is the sum of expected payoffs of assimilating agents. Note that an assimilating agent i of type 1 obtains the payoff of $\mu_R(i)$ with probability $\frac{1-y}{2}$, and so forth.

[15]Although this setup eliminates gains from trade, we can still draw some nontrivial conclusions.

In the autarky case, community A's total welfare is equal to $W^0 = \frac{mn}{m+n} + \frac{mn}{m+n} = \frac{2mn}{m+n}$. On the other hand, if $\mu_R(i) = \nu_R(i) = 1$ for all i, i.e., if agents in community A have no inherently preferred standard, then it is verified that integration leads to a completely assimilated equilibrium, (0,0), and the total welfare of community A is given by

$$\overline{W} = \int_0^m \frac{1}{2} di + \int_0^n \frac{1}{2} di = \frac{1}{2}(m+n). \tag{15}$$

The welfare difference between autarky and free trade with no inherently preferred standard is:

$$\overline{W} - W^0 = \frac{(m-n)^2}{2(m+n)} \geq 0,$$

where the strict inequality holds whenever $m \neq n$, i.e., each community has a comparative advantage in one good.

Under our assumption, agents in community A typically prefer standard L to standard R. Therefore, in the accessible equilibrium, the total welfare W^* is less than $\overline{W}$. Thus, gains (or losses) from integration can be expressed as:

$$W^* - W^0 = (W^* - \overline{W}) + (\overline{W} - W^0),$$

where the first bracket of the right hand side is the losses due to switching to a less preferable standard, and the second bracket corresponds to gains from trade. Note that the first bracket is always negative regardless of whether the accessible equilibrium is a completely assimilated or partially assimilated one. It is the relative size of these two effects that determines the overall welfare effect of integration.

To further examine the welfare consequences of integration, we now turn to the following explicit example.

Example 1. Uniform Distribution

Let $\mu_R(i)$ and $\nu_R(i)$ be distributed uniformly on (0,1), i.e., they are given by (3) and (4), respectively. We know from Section 3 that the stable equilibrium accessible from our initial (autarky) condition (m, n) is the completely assimilated equilibrium, (0,0).

From (15), total welfare for the type 1 members of community A at the equilibrium point is given by

$$W_1 = \frac{1}{2m} \int_0^m i \, di = \frac{m}{4}.$$

Similarly, total welfare for the type 2 members of community A is:

$$W_2 = \frac{1}{2n} \int_0^n i \, di = \frac{n}{4}.$$

Therefore the total welfare W^* is given by

$$W^* = \frac{m+n}{4}.$$

The amount of welfare change is, therefore, given by

$$\begin{aligned} W^* - W^0 &= (W^* - \overline{W}) + (\overline{W} - W^0) \\ &= -\frac{m+n}{4} + \frac{(m-n)^2}{2(m+n)} \\ &= \frac{1}{4(m+n)}\left[(m-n)^2 - 8mn\right]. \end{aligned} \tag{16}$$

Expression (16) is negative if the relative size of m and n is approximately between 0.1 and 10. If comparative advantage is not too strong one way or the other, total welfare decreases as the result of integration. Note that in this case, all agents in community B are better off, and the total welfare is increased by $\frac{(m-n)^2}{2(2-m-n)}$.

6 Welfare under Increasing-Returns-to-Scale Matching Technology

In the last section, we assumed that the matching technology exhibits constant returns to scale, i.e., no matter what the size of a community may be, the probability of an agent's matching with another is always one. This implies that gains from integration is limited to the standard gains from trade based on comparative advantages. When two communities are integrated, however, it is often the case that trade opportunities increase, i.e., the matching technology exhibits increasing returns to scale. In such a case, we have to modify our welfare analysis, including gains from expanding opportunities as a positive effect of integration.

We assume that the probability of matching is proportional to the size of the group, i.e., in the autarky case, the probability that an agent in community A (resp. B) is matched with another agent is $\frac{m+n}{2}$ (resp. $\frac{2-m-n}{2}$), while if the two communities are integrated, the probability of an agent's being matched with another is increased to one. With this additional benefit from integration, it is no longer true that the welfare decreases even if the degree of comparative advantage is small. Indeed, even in the case of $m = n$, it is verified that if $\mu_R(i) = i/m$ and $\nu_R(i) = i/n$ as in Example 1, the welfare increases as the result of integration since the welfare in the autarky case is now $\frac{1}{2}mn$, which is less than W^* for all m and n. This by no means implies that integration always leads to an increase the welfare of community A. In order to see this point, we now turn to the following specific examples. In particular, Example 3 shows that in the presence of three standards, it is possible that both communities are worse off after integration in spite of increasing returns to scale in the matching technology.

Example 2. Two Subgroups

We study the example in which all members of community A belong to one of two subgroups in terms of their inherent preferences: a fraction η of the type 1 agents in A have cost $\mu_R(i) = \overline{\mu}_R$ while the remaining $1-\eta$ of this population have cost $\mu_R(i) = \underline{\mu}_R$, where $\overline{\mu}_R > \underline{\mu}_R$; similarly, a fraction η of the type 2 agents in A have cost $\nu_R(i) = \overline{\nu}_R$ while the remaining agents of this type have cost $\nu_R(i) = \underline{\nu}_R$, where $\overline{\nu}_R > \underline{\nu}_R$.

We assume that $\underline{\mu}_R$, $\overline{\mu}_R$, $\underline{\nu}_R$, and $\overline{\nu}_R$ are such that, in equilibrium, players with costs equal to either $\overline{\mu}_R$ or $\overline{\nu}_R$ will switch to using standard R, while players with costs $\underline{\mu}_R$ or $\underline{\nu}_R$ will continue to use L. Specifically, this means that $\overline{\mu}_R > \frac{n}{1-n}$ and $\overline{\nu}_R > \frac{m}{1-m}$, while $\underline{\mu}_R < \frac{(1-\eta)n}{1-(1-\eta)n}$ and $\underline{\nu}_R < \frac{(1-\eta)m}{1-(1-\eta)m}$. From these conditions, we see that for a given $\overline{\mu}_R$ and $\underline{\mu}_R$, there is a range of n for which a partial equilibrium exists. An equivalent statement can be made for $\overline{\nu}_R$ and $\underline{\nu}_R$.

Given this, the total welfare of A's type 1 agents in equilibrium is now described by

$$W_1 = \frac{1}{2}\int_0^{(1-\eta)m} (1-\eta)n \, di + \frac{\overline{\mu}_R}{2}\int_{(1-\eta)m}^{m} (1-(1-\eta)n) \, di$$

which is equal to

$$W_1 = \frac{1}{2}(1-\eta)^2 mn + \frac{\eta}{2}\{\overline{\mu}_R[1-(1-\eta)n]m\}.$$

This expression, while somewhat complicated, is readily interpretable. The first term in the expression represents the utility level of the community members who continue to use L. This mass of members, in the autarky case, would have received welfare level

$$\int_0^{(1-\eta)m} \frac{n}{2} \, di = \frac{1}{2}(1-\eta)mn > \frac{1}{2}(1-\eta)^2 mn.$$

These members have suffered a welfare loss. This is, of course, the direct result of the negative externality imposed upon them when the $\overline{\mu}_R$ and $\overline{\nu}_R$ members of their community switch to R.

On the other hand, for their incentive constraint to have been satisfied, the agents with $\overline{\mu}_R$ and $\overline{\nu}_R$ must have experienced a welfare gain. This is easy to verify. The second term in the above expression represents the new level of welfare which these agents receive. Previously, again referring to the autarky case, they received

$$\int_{(1-\eta)m}^{m} \frac{n}{2} \, di = \frac{1}{2}\eta mn,$$

which, given our initial restrictions on $\overline{\mu}_R$ and $\overline{\nu}_R$, is strictly less than their new level of welfare.

	L	C	R
L	μ_L	0	0
C	γ	μ_C	γ
R	0	0	μ_R

Table 1: Three Standards

We now ask whether or not the welfare gain experienced by the agents using R outweighs the welfare loss incurred by those continuing to use L. We will look specifically at the case where $\overline{\mu}_R = \overline{\nu}_R = 1$, since if the inequality holds under this condition, it will certainly hold in the case where $\overline{\mu}_R$ and $\overline{\nu}_R$ are less than 1, as in this case, the welfare gain experienced by the gaining agents is diminished. As before, we look first at the net change in welfare which the type 1 agents experience, and then we can examine, separately, the type 2 agents. If the type 1 agents have incurred a net welfare loss, the following inequality will hold:

$$\eta(1-\eta)mn \;>\; \eta m[1-(1-\eta)n] \;-\; \eta mn,$$

which gives us the condition

$$n \;>\; \frac{1}{3-2\eta}$$

Since the relevant inequality for the type 2 agents is symmetric, we can conclude that the following condition will also hold if the type 2 agents experience a net welfare loss:

$$m \;>\; \frac{1}{3-2\eta}$$

Again, these conditions are sufficient but if $\overline{\mu}_R$ and/or $\overline{\nu}_R$ are strictly less than one, then weaker conditions will suffice. Either way, these conditions tell us immediately that for ranges of m and n, a net welfare loss may result with the expansion of trade opportunities when the costs of coordination are accounted for.

Example 3. Three Standards

We turn now to consider a situation in which agents may choose from among three behavioral standards, namely L, C and R. We look at a rather specific example. Let the payoff matrix for type 1 agents be that in Table 1. We assume that a corresponding matrix applies to the type 2 agents. However, as before, we focus our welfare analysis on the type 1 agents since the analysis for the 2 agents is symmetric.

We assume that the communities A and B are equal in size, i.e., $m = n = \frac{1}{2}$. We retain the assumption that the standard L is the most preferred standard by members of community A while R is most preferred by members of B. Thus, we retain the normalization that for type 1 members of A, $\mu_L(\cdot) = 1$, while for members of B, $\mu_R(\cdot) = 1$. Further, we retain the initial condition at (m, n), i.e., all members of A use L and all members of B use R. Note, however, that x and y as previous defined are no longer sufficient to characterize equilibrium here because of the addition of the third standard.

The welfare results in this case do not rely upon there being heterogeneity among agents with respect to μ_L and μ_R, though the results do hold for appropriate parameter values when heterogeneity is present. Thus, for ease of enumeration, we will assume that agents within a community are homogeneous in this regard. Further, we can assume that all agents in both communities earn the payoff γ when using C and trading with someone using L or R, while earning the payoff μ_C from using C and trading with someone using C.

We assume that $\gamma > max\{n, 1-n\}$ and that if agents from A and B switch to standard C from their preferred standards, they do so at the same rate.[16] We use the remainder of this section to show that if

$$\gamma > \frac{1}{2} > \mu_C > \frac{1}{2}\gamma - \frac{1}{4}, \tag{17}$$

then the lifting of a trade barrier between A and B leads to a welfare loss for every individual in both A and B.

Given our assumptions, the stable equilibrium accessible from our initial condition is the equilibrium in which all members of both A and B choose standard C. To see this, we first consider the decision of an agent at the initial point when the barrier is lifted. If the agent is a member of A, then taking L offers an expected payoff of $\frac{1}{2}n = \frac{1}{4}$, taking C offers $\frac{1}{2}\gamma$, and taking R offers $\frac{1}{2}\mu_R(1-n) = \frac{1}{4}\mu_R$. The agent's best/better response is clearly to choose C, given our assumption regarding the value of γ. The same argument holds for members of B. Thus, we expect some agents to switch to C.

Now, since agents from A and B switch to C at the same rate, we can say that at some fixed point in time, a fraction c of the agents in both groups are using C. Thus, in evaluating his options, an agent in A sees that his expected payoff equals $\frac{1}{2}(\frac{1}{2} - c)$ if he chooses L, $\frac{1}{2}[\gamma(1-2c) + \mu_C(2c)]$ if he chooses C, and $\frac{1}{2}\mu_R(\frac{1}{2} - c)$ if he chooses R. Since choosing R is clearly a dominated strategy, we need only assess the comparison between his choosing L and C. Doing so, we find that if the following equation holds, then an agent will still prefer C to L if the following inequality holds.

$$\gamma(1-2c) + \mu_C(2c) - (\frac{1}{2} - c) > 0$$

This equation will hold for all $c \in [0, 1]$ if it holds for c=1. Thus, we find that if

$$\mu_C > \frac{1}{2}\gamma - \frac{1}{4}$$

[16] One assumption which we could make that would make our assumption regarding agents switching at the same rate most intuitively appealing is the assumption that μ_R for agents in A is equal to μ_L for agents in B. However, as this assumption is in itself not necessary, we do not make it.

then in equilibrium, all agents will stay with the choice C. In equilibrium, the payoff expected by every agent equals $\frac{1}{2}\mu_C$. If $\mu_C < \frac{1}{2} = n = 1 - n$, then the expected payoff to every agent is lower than it was at the initial condition. Therefore, we say that all agents in both communities experience a welfare loss upon the lifting of the trade barrier between the communities.

7 Concluding Remarks

We have highlighted the importance of explicitly considering the need for coordination in interactions when modeling economic behaviors where such coordination is required. We have shown that when we account for the costs of such coordination, there are cases in which total welfare of a minority community decreases when a trade barrier between the two communities is lifted. In addition, we offered an example which illustrates that in a situation where no dominant culture exists, every member of both communities may ultimately be worse off upon the lifting of a trade barrier.

A few remarks are in order. First, we have not in this paper considered important intergenerational issues which are pertinent in any discussion of assimilation. It is often argued that one of the most serious problems associated with assimilation is the gap which arises between generations. Parents become alienated from their children and cannot pass on the wisdom they have inherited from generations of people that came before them. Children who wish to assimilate must learn the new culture on their own. They often remain second class citizens in the new society. This effect may persist, in some cases becoming intensified and while in others, becoming weaker. In cases where this effect becomes larger, the rate of economic growth may be higher for members of a dominant group in society than for those coming from a minority group. In addition to this problem, we have assumed that people make their choices myopically. We have not considered the case in which people take into account future generations when making their own decisions regarding assimilation.

Our next remark is related to our first. We do not presently deal with situations in which discrimination makes it essentially impossible for one group to coordinate with, or assimilate into, another group. This problem arises most commonly in cases when a group has some recognizable traits which cannot be changed, even by choice, such as gender or skin color. As the Folk Theorem has shown us, discrimination is sustainable in equilibrium even if the only difference between people is their "names." In such cases, it may be that members of one community would like to coordinate with the members of another group, but when they take the appropriate behaviors which would seemingly allow them to do so, they effectively end up as a group unto themselves, forced to interact primarily within the newly formed, third group.

Third, if some people can switch between two standards, these people may act as middlemen between the two communities. Examples are international merchants and English-speaking Chinese-Americans in Chinatown. In these cases, while the minority may not lose their identities, the wealth could be concentrated on a handful middlemen.

Fourth, different situations present different problems. For example, in the case of computer networks, standardization may imply the need for complete coordination. On the other hand, culture cannot be described by a single trait (Cavalli-Sforza and Feldman (1981)). Adopting one trait but not another may have effects which we do not capture with the model in this paper.[17] Thus, we must more carefully examine the contents of such traits when we apply our analysis to specific problems. One typical question which must be addressed in this vein is the question or which traits can be changed and at what cost.

As can be seen, the model presented in this paper is far from the universal one. We only suggest one of many possibilities. Still, it raises an issue that has been ignored in the literature. If the reader realizes that the demand for coordination sometimes offsets gains from trade, a half of the goal of this paper would be achieved.

References

[1] Akerlof, George A., and Rachel E. Kranton (2000), "Economics and Identity," *Quarterly Journal of Economics* 115, pp. 715-753.

[2] Asimov, Stanley, ed. (1995), *Yours, Isaac Asimov: A Lifetime of Letters* (New York: Doubleday).

[3] Baldwin, Richard E., and Anthony J. Venables (1995), "Regional Economic Integration," *Handbook of International Economics, vol. III*, eds. by G. Grossman and K. Rogoff (Elsevier Science B.V.): pp.1597-1644.

[4] Cavalli-Sforza, Luigi L. and Marcus W. Feldman (1981), *Cultural Transmission and Evolution: A Quantitative Approach* (Princeton, NJ: Princeton University Press).

[5] Cose, Ellis (1993), *The Rage of a Privileged Class* (New York: HarperCollins Publishers).

[6] de Beauvoir, Simone (1952), *The Second Sex* (New York: Alfred A. Knopf, Inc.), pp. 679-715.

[7] Gale, Douglas (1986), "Bargaining and Competition Part I: Characterization," *Econometrica*, 54, pp.785-806.

[17] Matsui and Okuno-Fujiwara (2002) study this point to show that eclectic culture may emerge after integration.

[8] Gilboa, Itzhak and Akihiko Matsui (1991), "Social Stability and Equilibrium," *Econometrica* 59, pp.859-867.

[9] Hart, Oliver (1975), "On the Optimality of Equilibrium When the Market Structure is Incomplete," *Journal of Economic Theory,* pp. 418-443.

[10] Hofbauer, Josef and Bill Sandholm (2002), "On the global convergence of stochastic fictitious play," *Econometrica* 70, pp.2265-2294.

[11] Kandori, Michihiro (1997), "Evolutionary Game Theory in Economics," *Advances in Economics and Econometrics: Theory and Applications,* eds. by David M. Kreps and Kenneth F. Wallis (Cambridge University Press): pp.243-277.

[12] Lazear, Edward (1999), "Culture and Language," *Journal of Political Economy*, 107: pp.95-126.

[13] Matsui, Akihiko and Masahiro Okuno-Fujiwara (2002), "Evolution and the Interaction of Conventions," *Japanese Economic Review*, 53, pp.141-153.

[14] Matsuyama, Kiminori, Nobuhiro Kiyotaki and Akihiko Matsui (1993), "Toward a Theory of International Currency," *Review of Economic Studies*, 60, pp.283-307.

[15] Ray, Debraj (1998), *Development Economics* (Princeton University Press).

[16] Rubin, Barry (1995), *Assimilation and Its Discontents* (New York: Times Books).

[17] Rubinstein, Ariel. and Wolinsky, Asher. (1985),"Equilibrium in a Market with Sequential Bargaining, " *Econometrica* 53, pp.1133-50.

[18] Schlag, Karl (1998), "Why imitate, and if so, how? A bounded rational approach to multi-armed bandits," *Journal of Economic Theory*, 78, pp. 191-209.

[19] Smith, Hal (1988), "Systems of Ordinary Differential Equations Which Generate an Order Preserving Flow. A Survey Results," *SIAM Review*, 30, pp.87-113.

[20] Taylor, Peter and Leo Jonker (1978), "Evolutionarily stable strategies and game dynamics," *Mathematical Biosciences*, 40, pp. 145-156.

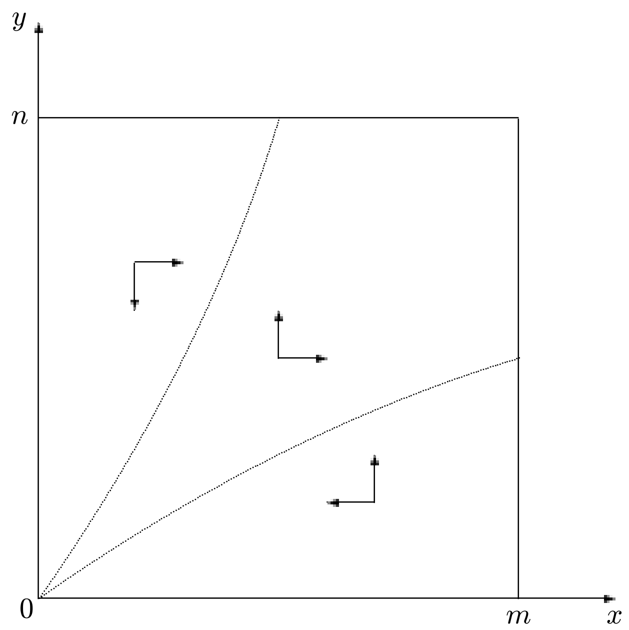

Figure 1: Equilibrium: Autarky with Uniform Distribution

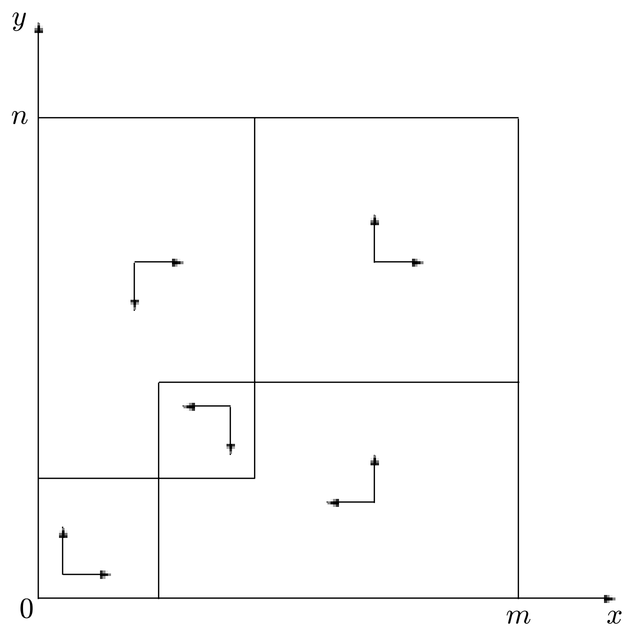

Figure 2: Equilibrium: Autarky with Two Mass points

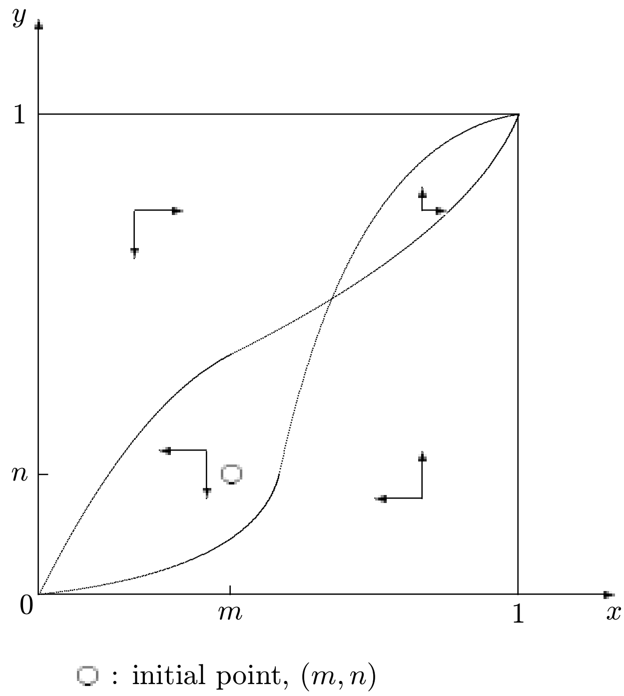

Figure 3: Phase Diagram: Uniform Distribution

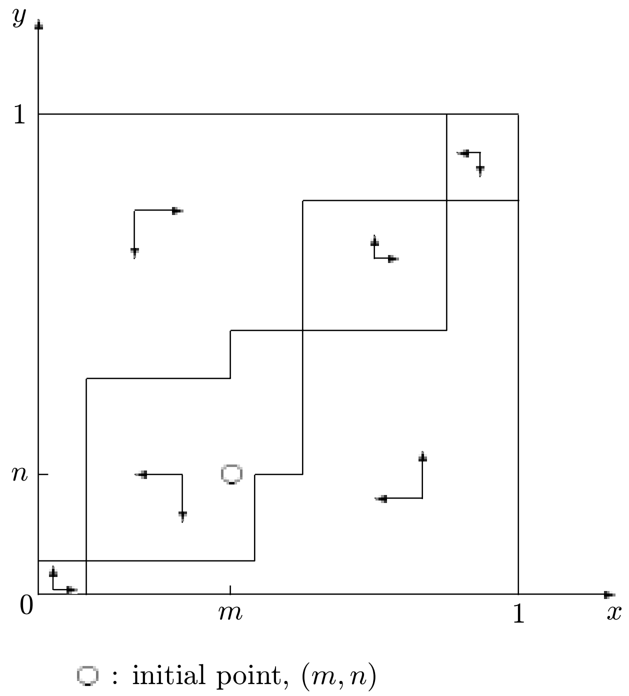

Figure 4: Phase Diagram: Two Mass Points for Each Community

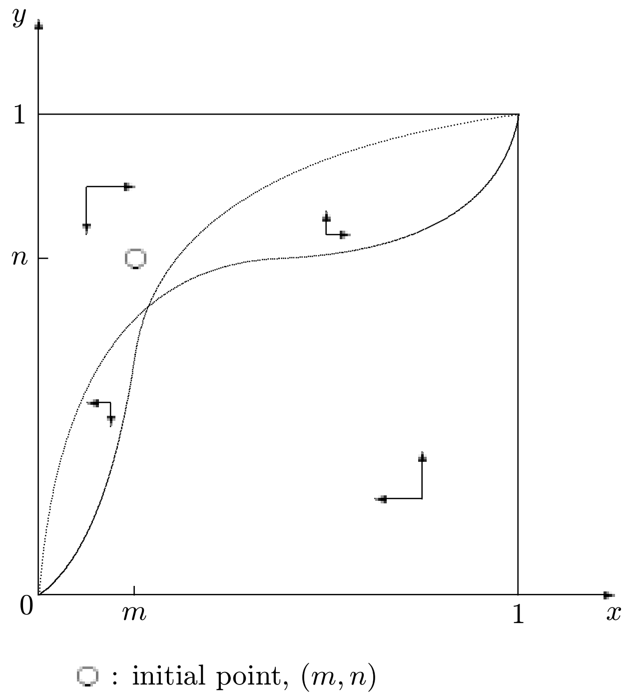

Figure 5: Incomparable Community Sizes