Mathematics and Economic Modeling
Problem set #1

Summer 2014(ay)
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No need to hand in. Use definitions with and without a prime (') as defined in class.

1. Prove the following statements in $\mathbb{R}^1$.

(a) $(a, +\infty)$ is open.

(b) $\{a\}$ is closed.

(c) $[a, b]$ $(a < b)$ can be expressed as an intersection of countable collection of open intervals.

(d) (a, b) $(a < b)$ can be expressed as a union of countable collection of closed intervals.

(e) (x_n) with $x_n = 1/n$ converges to 0.

(f) Given $S \subset \mathbb{R}^1$, $u \in \mathbb{R}^1$ is the supremum of S if and only if (i) $\nexists x \in S(u < x)$, and (ii) $v < u \rightarrow (\exists x \in S(v < x))$.

2. Prove the following properties of closed sets in $X = \mathbb{R}^n$.

(a) $\emptyset, X$ are closed.

(b) the intersection of any collection of closed sets is closed.

(c) the union of any finite collection of closed sets is closed.

3. Show the equivalence between convergence and convergence' in $\mathbb{R}^n$.

4. Show the equivalence between closedness and closedness' in $\mathbb{R}^n$.

5. Show that a set F is closed if and only if it contains all of its boundary points in $\mathbb{R}^n$.

6. Show that every closed set can be expressed as an intersection of a countable collection of open sets in $\mathbb{R}^n$.

7. Give an example of a sequence of nonempty sets (I_n) that are bounded above and below such that $I_{n+1} \subset I_n$, but their intersection is empty.

8. Give an example of a sequence of closed nonempty sets (J_n) such that $J_{n+1} \subset J_n$, but their intersection is empty.