Mathematics and Economic Modeling
Problem set #2

Summer 2014(ay)
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No need to hand in.

1. Prove the following theorem.

Theorem 1 *(Lebesgue Covering Theorem) Suppose that* $\mathcal{S} = \{S_\lambda\}_{\lambda \in \Lambda}$ *is an (open) covering of a compact set* $F \subset \mathbb{R}^K$. *Then there exists* $\alpha > 0$ *such that if* $x, y \in F$ *and* $d(x, y) < \alpha$, *then there exists* $S_\lambda \in \mathcal{S}$ *with* $x, y \in S_\lambda$.

2. Prove the following statements in $X = \mathbb{R}^K$.

(a) $(0, 1)$ $(K = 1)$ is not compact without resorting to the Heine-Borel theorem.

(b) a compact set is closed.

(c) a compact set is bounded.

3. Show the equivalence of the following three conditions for a correspondence $F : X \to\to Y$ where both X and Y are subsets of the Euclidean spaces:

1. F is lhc.
2. $F_+^{-1}(T)$ is closed for every T that is closed in Y.
3. $F^{-1}(S)$ is open for every S that is open in Y.

4. Let $F : [0, 2] \to\to [0, 1]$ be a correspondence.

(a) Suppose that F is given by

$$F(x) = \begin{cases} \{0\} & \text{if } x < 1 \\ \texttt{[0,1]} & \text{otherwise.} \end{cases}$$

Is F uhc, lhc, or both? Answer with a proof.

(b) Suppose that F is given by

$$F(x) = \begin{cases} \{0\} & \text{if } x \leq 1 \\ \texttt{[0,1]} & \text{otherwise.} \end{cases}$$

Is F uhc, lhc, or both? Answer with a proof.