Mathematics and Economic Modeling
Problem set #3

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No need to hand in.

1. Give the definitions of the following concepts.

(a) open set in the Euclidean space by using the Euclidean metric.

(b) closed set

(c) compact set (topological definition)

(d) continuity of a function (topological definition)

(e) upper hemicontinuity of a correspondence

(f) metric space

2. Show that a Cauchy sequence in the Euclidean space is convergent.

3. Construct an example of a metric space of the form $(\mathbb{R}, \rho)$ in which some Cauchy sequence is *not* convergent (note that the set itself is the same as the one in the Euclidean space, but the metric is not).

4. In a game in strategic form, $\langle I, (A_i)_{i \in I}, (u_i)_{i \in I} \rangle$, show that the best response correspondence

$$BR_i(\sigma_{-i}) = \{\sigma_i \in \Delta(A_i) | \ \forall \sigma_i' \in \Delta(A_i)[u_i(\sigma_i, \sigma_{-i}) \geq u_i(\sigma_i', \sigma_{-i})]\}$$

is upper hemicontinuous and convex-valued, where $\Delta(A_i)$ is the set of mixed strategies of player $i \in I$, and u_i is the expected payoff function of player i.

5. Show that the budget set correspondence $B(p) = \{x \in \mathbb{R}_+^K | \ p \cdot x \leq M\}$ in the standard consumer theory is continuous in (p, M) on $(0, \infty)^{K+1}$.